Lecture Lecture 45 real video *53 min*. Consider the hyperplanes bisecting the edges of a matroid polytope. The reflection group generated by the reflections across these hyperplanes is finite - it is a product of symmetric groups. This is an extremely rare phenomenon: the (irreducible) finite reflection groups can be classified into four infinite families and six exceptional examples. These objects appear in many different fields of mathematics. This suggests the ongoing study of "Coxeter matroid polytopes" and "Coxeter matroids" - a larger family which includes all matroid polytopes and all regular polytopes as examples.

Lecture Lecture 44 real video *54 min*. We characterize the facets of a matroid polytope. We show that the matroid polytopes of M/i and M\i are the faces x_i=1 and x_i=0 of the matroid polytope of M. We show that a matroid and its dual have congruent polytopes. Conversely, a matroid polytope essentially determines the matroid up to duality, loops, and coloops.

Lecture Lecture 43 real video *52 min*. We define the connected components of a matroid and show that the codimension of the matroid polytope P_M is the number of components of M.

Lecture Lecture 41 real video *51 min*. We present the inequality description of a matroid polytope, and use linear programming duality to prove it.

Lecture Lecture 40 real video *51 min*. We prove that every basis of M gives a vertex of P_M, and present (without proof) the inequality description of P_M. We conclude with an explanation of linear programming duality.

Lecture Lecture 39 real video *52 min*. After a crash course on polytopes, we define the matroid polytope P_M of a matroid M.

Lecture Lecture 38 real video *52 min*. We give an interpretation of the coefficients of the Tutte polynomial. This gives a nice formula for the Tutte polynomial of the Catalan matroid. As a consequence we get results about the combinatorics of Dyck paths.

Lecture Lecture 37 real video *51 min*. A knot is an embedding of a circle in R^3. It can be represented by a knot diagram on the plane D, or by a signed planar graph S(D). Two knot diagrams D and E represent the same knot if and only if one can get from D to E via Reidemeister moves. To prove that two knot diagrams represent different knots, one uses knot invariants such as the Jones polynomial. The Jones polynomial of an alternating knot D is an evaluation of a Tutte poynomial of the graph S(D).

Lecture Lecture 36 real video *50 min*. We define the Hamming code, which is the largest possible code that can detect AND correct one error.

Lecture Lecture 35 real video *52 min*. Error-correcting codes allow us to transmit messages over noisy channels. Linear codes are very useful ones, and their weight enumerator is given by a Tutte polynomial. MacWilliams's identity gives a powerful relation between a linear code and its dual; it follows from matroid duality.

Lecture Lecture 34 real video *52 min*. The probability of reliability of a network is an evaluation of its Tutte polynomial. If M and N are matroids dual to each other, then T_M(x,y) = T_N(y,x). Tutte polynomials can be computed by solving an enumerative problem over a finite field.

Lecture Lecture 33 real video *52 min*. The Tutte polynomial T_M of a matroid M easily determines: the number of bases, of independent sets, of spanning sets, of elements, the rank, and the characteristic polynomial of M. The Tutte polynomial of a graph determines its chromatic and flow polynomials.

Lecture Lecture 32 real video *51 min*. Many interesting invariants of a matroid are called "Tutte-Grothendieck invariants" because they satisfy the same recursive formula. These are all evaluations of one T-G invariant, which is "the mother of all T-G invariants": the Tutte polynomial T_M(x,y). We discuss how you can compute this polynomial recursively or directly.

Lecture Lecture 31 real video *53 min*. The finite field method is a very useful way to compute characteristic polynomials of arrangements. We prove it and apply it to several arrangements.

Lecture Lecture 30 real video *52 min*. We give two proofs of Zaslavsky's theorem. The first one is based on the deletion-contraction recursion. The second one is a topological argument, based on computing Euler characteristics.

Lecture Lecture 29 real video *54 min*. We give a formula for the Moebius function of the lattice of flats of a matroid. We obtain Whitney's formula for the characteristic polynomial of a matroid. We use it to prove the deletion-contraction formula for characteristic polynomials of matroids.

Lecture Lecture 28 real video *53 min*. We prove the Moebius inversion formula. We state Zaslavsky's theorem for the number of regions (and bounded regions) of a hyperplane arrangement in a real vector space.

Lecture Lecture 27 real video *54 min*. We compute the Moebius function of the lattice of flats of a uniform matroid. We define the two-variable Moebius function of a poset, and state the Moebius inversion formula. We see how this generalizes the Moebius inversion formula of number theory, and the inclusion-exclusion formula of combinatorics. (Note. The microphone is off for the first 20 minutes, sorry...)

Lecture Lecture 26 real video *51 min*. We state some known and unknown facts about algebraic matroids. We then start a new topic: enumeration in matroids. We define the Moebius function and characteristic polynomial of a graded poset. We discuss their relationship with counting proper colorings of a graph.

Lecture Lecture 24 real video *52 min*. We prove Desargues's theorem, and use it to construct another non-linear matroid: the non-Desargues matroid. We define algebraic matroids.

Lecture Lecture 23 real video *53 min*. We discuss the problem of characterizing the matroids that are linear over (at least one)/(a fixed)/(every) field. We see two matroids which are not linear. We discuss the affine representation of a linear matroid.

Lecture Lecture 22 real video *53 min*. We can prove a matroid is not graphical by finding a non-graphical matroid "living inside it" (as a minor). Tutte proved there are exactly five obstructions to being graphical. We knew three of them already, and we discuss the other two: the Fano matroid and its dual.

Lecture Lecture 21 real video *51 min*. We prove that minors of linear matroids are linear. Minors of transversal matroids aren't necessarily transversal. Gammoids are the smallest class which contains transversal matroids, and is closed under minors and duals.

Lecture Lecture 20 real video *44 min*. We describe the rank function of a dual matroid. We see how deletion and contraction work for graphical and linear matroids. We conclude that minors of graphical matroids are graphical, and minors of linear matroids are linear.

Lecture Lecture 19 real video *50 min*. We see how to view a matroid pictorially as a rank function on a hypercube. We use this to define deletion, contraction, and minors.

Lecture Lecture 18 real video *53 min*. We finish the proof that geometric lattices are in one-to-one correspondence to simple matroids. We prove that geometric lattices are coatomic.

Online lecture videos

Lecture Lecture 17 real video *53 min.* A lattice is geometric if and only if it is the lattice of flats of a matroid.

Lecture Lecture 16 real video *53 min.* The lattice of flats of a matroid is a lattice, it is graded, it is semimodular, and it is atomic.

Lecture Lecture 15 real video *53 min.* We define posets and lattices and study some examples. We define the lattice of flats of a matroid.

Lecture Lecture 14 real video *50 min.* We prove that matroids satisfy the four closure axioms, and state that these axioms characterize matroids.

Lecture Lecture 13 real video | Notes *53 min.* We define the rank function of a matroid, prove the rank axioms, and sketch a proof that they give another characterization of matroids. We define the closure operator of a matroid and state the closure axioms. (Note. Unfortunately the motor on the camera is off for the first 19:40 minutes. Follow the audio with the lecture notes "lecture13a" and "lecture13b" in the link above.)

Lecture Lecture 12 real video *53 min.* We define routings in a directed graph. We use them to define cotransversal matroids, and state that they are precisely the duals of transversal matroids. We define the intersection poset of a hyperplane arrangement.

Lecture Lecture 11 real video *50 min.* We prove that graph duality is an instance of matroid duality: If G is a plane graph and G* is its dual, then M(G*) = M(G)*. We get a proof of Euler's formula v-e+f=2. We state Whitney's 2-isomorphism theorem, which describes when two different graphs have the same matroid.

Lecture Lecture 10 real video *53 min.* Matroid duality generalizes graph duality. We introduce the dual of a plane graph and its properties. We discuss the Four Color Theorem.

Lecture Lecture 9 real video *53 min.* For linear matroids, duality is orthogonality. We describe the matroid of a subspace V of a vector space. (A better description is in the lecture notes on the course website.) If M is the matroid of V, we prove that the dual matroid M* is the matroid of the orthogonal complement of V.

Lecture Lecture 8 real video *53 min.* We prove that dual matroids are in fact matroids. We discuss what duality means for linear and graphical matroids.

Lecture Lecture 7 real video *53 min.* We prove (one direction of) the equivalence between the axiom systems for circuits and independent sets. We define (without proof) dual matroids. We define fundamental circuits, and prove an alternative basis exchange lemma.

Lecture Lecture 6 real video *53 min.* After some comments on axiom systems in general, we define the circuits of a matroid, and propose an axiom system for them.

Lecture Lecture 5 real video *53 min.* We introduce the axiom system for the collection of bases of a matroid, and prove that it is equivalent to the axiom system for independent sets.

Lecture Lecture 4 real video *53 min.* We present, without proof, Kirchhoff's matrix tree theorem for the number of spanning trees of a graph. We show that the basis of minimum weight of a matroid can be found using the greedy algorithm. In fact, matroids are the simplicial complexes where this algorithm works.

Lecture Lecture 3 real video *52 min.* We define bases of a matroid and show they have the same size. We discuss what this means for linear, graphical, and transversal matroids.

Lecture Lecture 2 real video *52 min.* We prove that vector configurations, graphs, and matching problems give rise to three families of matroids called linear, graphical, and transversal.

Lecture Lecture 1 real video *53 min.* After discussing administrative aspects, I introduce some motivating examples for the course. We discuss three problems in linear algebra, graph theory, and matching theory which are superficially different, but are very closely related.

Course advertisement real video *14 min.*