D_n: $x_i = \pm x_j$ $\quad 1 \le i < j \le n$

$\to \chi_{D_n}(q) = \#\{(x_1,\dots,x_n) \in \mathbb{F}_q^n : x_i \neq \pm x_j\}$

- no zeroes: $(q-1)\cdots(q-2n+1)$ (as in B_n)
- with zeroes (exactly one): n choices for which coord is 0; $(q-1)\cdots(q-2n+3)$ (as in B_{n-1})

$$\chi_{D_n}(q) = [(q-1)\cdots(q-2n+3)][(q-2n+1)+n]$$

$$\chi_{D_n}(q) = (q-1)\cdots(q-2n+3)(q-n+1)$$

$$r(D_n) = 2\cdot 4\cdot\ldots\cdot(2n-2)\cdot n = 2^{n-1}\cdot n!$$

E_6, E_7, E_8: messier equations, but answers are very nice, too.

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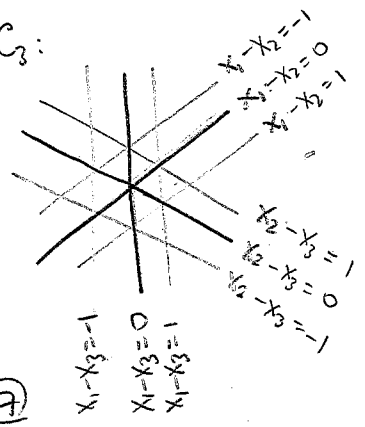
Other nice arrangements

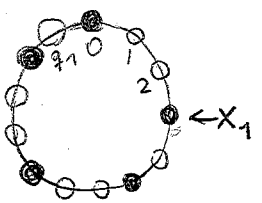
Catalan arrangement:

C_n: $\quad x_i - x_j = -1, 0, 1 \quad 1 \le i < j \le n$

C_3:

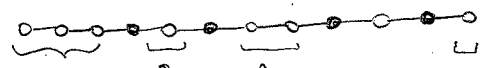


$\chi_{C_n}(q) = \#$ of $(x_1,\dots,x_n) \in \mathbb{F}_q^n$ such that $|x_i - x_j| > 1$



Choose $x_1 \to q$ possibilities

Then "unwrap" $\mathbb{F}_q - x_1$:



(no two consecutive •s)

To assign $x_2,\dots,x_n$,

- choose $a_1, a_2, \dots a_n > 0$ with $a_1 + \dots + a_n = q-n$ $\quad \binom{q-n-1}{n-1}$
- assign $x_2 \dots x_n$ to the $n-1$ •s $\quad (n-1)!$

So

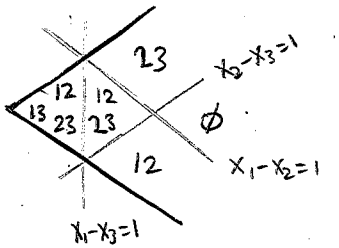
$$\chi_{C_n}(q) = q\binom{q-n-1}{n-1}(n-1)! = q(q-n-1)(q-n-2)\cdots(q-2n+1)$$

and

$$r(C_n) = 1\cdot(n+2)(n+3)\cdots(2n) = \frac{(2n)!}{(n+1)!} = n!\,C_n$$

(C_n: Catalan number)

So in each region of the braid arrangement there are C_n regions of C_n! Bijection?



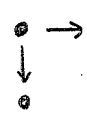
Label each region with the pairs ij with $x_i - x_j < 1$

Note: $x_i - x_j < 1 \to x_{i+1} - x_j < 1$

$\downarrow$

$x_i - x_{j-1} < 1$

Put a dot on each such ij, so



Then these are in bijection with Dyck paths! So there are C_n of them