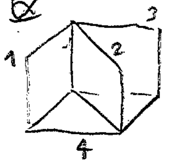
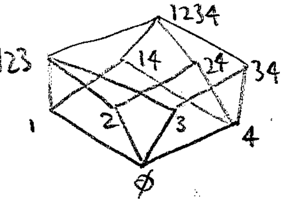
$\mathcal{A}$-hyp arr in $\mathbb{R}^d$.

Def The <u>intersection poset</u> $L_\mathcal{A}$ of $\mathcal{A}$ has:
elements: intersections of subsets of $\mathcal{A}$ ("flats")
order: $F < G$ if $F \supset G$

<u>Ex</u>

$\mathcal{A}=$ 

$L_\mathcal{A}=$ 

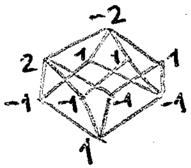
label $H_i \cap \dots \cap H_j$ with set $\{i, \dots, j\}$

<u>Note</u> This has a meet: $F \wedge G = F \cap G$
It has a join if it has a $\hat{1} = \bigcap_{H \in \mathcal{A}} H$
So: $\boxed{\begin{array}{l} \Rightarrow L_\mathcal{A} \text{ meet semilattice} \\ \mathcal{A} \text{ central} \Rightarrow L_\mathcal{A} \text{ lattice} \end{array}}$

Also: $\boxed{L_\mathcal{A} \text{ graded}}$ $\quad \text{rank}(F) = d - \dim F$

Let P be a poset.
The <u>Möbius function</u> of P is:

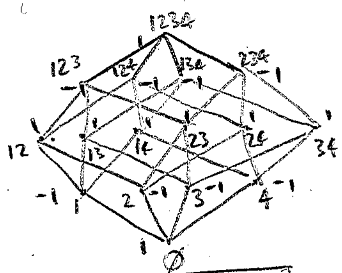
$$\mu(x) = \begin{cases} 1 & \text{if } x \text{ is minimal} \\ -\sum_{y<x} \mu(y) & \text{otherwise} \end{cases}$$

Ex: 

$\longrightarrow \chi_\mathcal{A}(q) = q^3 - 4q^2 + 5q - 2$

The <u>characteristic polynomial</u> of $\mathcal{A}$ is

$$\boxed{\chi_\mathcal{A}(q) = \sum_{F \in L_\mathcal{A}} \mu(F)\, q^{d - \dim F}}$$

Ex: $P = B_4$ 

By induction, $\boxed{\mu(S) = (-1)^{|S|}}$:

- $\mu(\emptyset) = 1$ ✓
- $\mu(S) = -\sum_{T<S} \mu(T) = -\sum_{i=0}^{k-1} \binom{k}{i}(-1)^i = (-1)^k$ $\quad (|S| = k)$

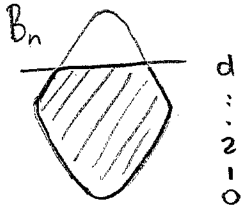
ind.

$(1-1)^k = 0$

<u>Ex</u> $\mathcal{A} = \{H_1, \dots, H_n\}$ in $\mathbb{R}^d$ generic.

$$\dim(H_{i_1} \cap \dots \cap H_{i_k}) = \begin{cases} d-k & k \le d \\ 0 & k > d \end{cases}$$

So

$L_\mathcal{A} =$ 

So $\boxed{\chi_\mathcal{A}(q) = q^d - \binom{n}{1} q^{d-1} + \binom{n}{2} q^{d-2} - \dots + (-1)^d \binom{n}{d} q^0}$

<u>Theorem</u> (Zaslavsky, 1975)
$\mathcal{A}$ arrangement $\Rightarrow$ $r(\mathcal{A}) = (-1)^d \chi_\mathcal{A}(-1)$
$b(\mathcal{A}) = (-1)^d \chi_\mathcal{A}(1)$

<u>Corollary</u>.
$\mathcal{A}$ generic $\Rightarrow$ $r(\mathcal{A}) = \binom{n}{d} + \binom{n}{d-1} + \dots + \binom{n}{1} + \binom{n}{0}$
$b(\mathcal{A}) = \binom{n}{d} - \binom{n}{d-1} + \dots + (-1)^d \binom{n}{0}$.