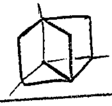
This course is an introduction to:

lecture 1
Aug 25.

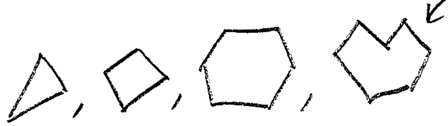
(convex) polytopes   hyperplane arrangements



with a combinatorial focus.

POLYTOPES

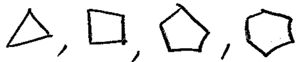
2-D Polygons:



not convex

Def A convex polygon P is one where: if $x, y \in P$ then the line segment xy is in P.

Combinatorial types of polygons:

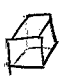


(boring)

$v=3$ $v=4$
$e=3$ $e=4$

Nice representatives: regular n-gons ($n=3,4,5,\ldots$)

3-D Polytopes:

		v	e	f	v	e	f		
tetrahedron		4	6	4	20	30	12	Google	dodecahedron
cube		8	12	6	12	30	20	Google	icosahedron
square pyramid		5	8	5	60	90	32		truncated icosahedron
octahedron		6	12	8					

Combinatorial types are much more complex. But there is order:

Theorem (Euler, 1752)
$$v-e+f=2$$

← Klee: "first landmark in th. of polytopes"
Alexandroff-Hopf: "first important event in topology"

In fact,

Theorem (Steinitz, 1906)
$\exists$ 3-polytope with
v vertices
e edges
f faces
$\iff$ $v-e+f=2$, $v \leq 2f-4$, $f \leq 2v-4$ (HW)

Does this characterize the combinatorial types of 3-polytopes?

Question Do there exist combin. distinct polytopes with the same number of vertices, edges, faces? (HW)

Also, very few combin. types have a regular realization.

Only regular polytopes: tetrahedron, cube, octahedron, dodecahedron, icosahedron

4-D "Polychora":

- Description of combin. types: probably hopeless.
- There is an analog to Euler's theorem
- There is no known analog to Steinitz's theorem

  Ziegler + team have worked hard on this and made much progress. It is much harder, and not done (yet?)!

- Only regular polychora: simplex, cube, crosspolytope, 24-cell, 120-cell, 600-cell