4. Here we try to exploit an idea from the class. We will do the following construction, which i will describe it very informally. Consider a polytope P and its prism $P \times I$ and consider one facet F of P and contract $F \times I$ to $F \times \{0\}$, that is, to the original face F. At the same time tilt the upper facet, $P \times 1$ in order that it is still a facet (a hyplerplane). This is called the wedge of P over the facet F, and lets called the result P'. So this is almost the same thing as the prism but one of the vertical facets is contracted to a ridge. This construction obviously is in one dimension higer, and has one more facet (que prism has two more, but we deleted one).

   Any vertex on the upper facet is joined by an edge with one just one vertex in the lower facet (that edges are precisely the results of $\{v\} \times I$). So name the vertices of $P \times \{0\}$ $v_1, \cdots, v_l$ and the corresponding ones in the upper facet by $v_1', \cdots, v_l'$ (there will be a few in which $v_i' = v_i$). Consider the shortest path from v to v' in P. Take any path from v to v', consider the sequence of edges $e_1, \cdots, e_k$. Now ignore all the edges that are of the form $v_i v_i'$, and if $e_j = v_m' v_n'$ replace it by $e_j' = v_m v_n$. In this way any path from v to v' in this malformed prism, gives us a path in the original polytope. This shows that any path is at least as long as the shortest. This ideas are resumed in the inequality $\Delta(d, n) \leq \Delta(d+1, n+1)$. Finally in a completely analogous way as in the third point, but now going up, by succesively applying the inequality we arrive at $\Delta(d, n) \leq \Delta(n-d, 2(n-d))$.