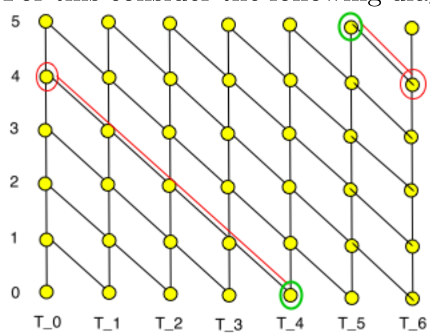
5. (Joint work with Diego Cifuentes and Fabian Prada)

- We construct T^{-1} reversing from top to bottom: Given a flag $\mathbb{F}$ construct the inverse recursively in the following way $T^{-1}F(-1) = \emptyset$, $T^{-1}F(d) = P$ and, for each $i > 0$, $T^{-1}F(d-i)$ is the unique face in dimension $d-i$ other than $F(d-i)$ which is contained in $T^{-1}F(d-i+1)$ and contains $F(d-i-1)$.
- For this consider the following diagram called *yapi diagram* (courtesy of Fabian Latorre)



The y axis represent the dimension of the face (we ignore dimension -1 and d, in this case 6, because all the flags are equal there), and the x axis represent how many times we have applied T to the initial flag in the first column. The lines represent containment if a point is above another. A funny thing to note is that the diagonals are also flags, and the action of T move them to the left, contrary to what it does to our normal flags. Lets say the point (i, j) is the one in column i row j.

Lemma (i, j) is not contained in $(i+1, k)$ for $j \leq k$
Proof By contradiction suppose $(i, j) \subset (i+1, k)$. If $j < k$ then, since $(i+1, j) \subset (i+1, k)$, by definition $(i+1, k)$ should contain the joint of those two points $(i, j+1)$. Doing this successively we arrive to the case $j = k$, so that $(i, k) \subset (i+1, k)$, but this is clearly impossible since $(i+1, k) \subset (i+1, k)$ and by the same argument $(i+1, k)$ would contain $(i, k+1)$, absurd since it has more dimension.

Now we are ready to prove that in the *yapi diagram* there can not be two equal elements in the same row. Suppose we have two equal points $(i, k) = (l, k)$, with $i < l$. Then the idea is that with (i, k) we can slide down through a diagonal and in (l, k) slide up until we get a situation described by the lemma, hence getting a contradiction.

Each time you go one step down on the diagonal of (i, k) or one up in the diagonal of (l, k) the columns of the points are getting one unit closer. And we can do this k times going down and $d-k-1$ going up, so we can reduce the diference of the columns in $d-1$, but we are supposing that $l-i \leq d$, so we can always bring that difference to one and get a contradiction by the lemma.

- T is weird