3)Lets prove that convex $\{+1, -1\}^d = \{x \in \mathbb{R}^d : -1 \le x_i \le 1, \text{ for all } 1 \le i \le d\}$:

I will check first convex $\{+1, -1\}^d \subseteq \{x \in \mathbb{R}^d : -1 \leq (x)_i \leq 1, \text{ for all } 1 \leq i \leq d\}$: Let $v_1, v_2, ..., v_{2^d}$ be any order of the points in the set $\{+1, -1\}^d$. Let $x \in \text{convex}\{+1, -1\}^d$, so we

write $x = \lambda_1 v_1 + \lambda_2 v_2 + \ldots + \lambda_{2d} v_{2d}$, where $\lambda_1 + \lambda_2 + \ldots + \lambda_{2d} = 1$, and $\lambda_j \ge 0$ for all j. Observe that $(x)_i = (\lambda_1 v_1 + \lambda_2 v_2 + \ldots + \lambda_{2d} v_{2d})_i = \lambda_1 (v_1)_i + \lambda_2 (v_2)_i + \ldots + \lambda_{2d} (v_{2d})_i$; we know that $(v_k)_i \in \{+1, -1\}$ for $1 \le k \le 2^d$, so its also true that $-1 \le (v_k)_i \le 1$. Since $\lambda_k \ge 0$, using the previous inequality, we get $-\lambda_k \le \lambda_k (v_k)_i \le \lambda_k$, for all k. Addind this inequalities we get $-(\lambda_1 + \lambda_2 + \ldots + \lambda_{2d}) \le (x)_i \le (\lambda_1 + \lambda_2 + \ldots + \lambda_{2d}) \Rightarrow -1 \le (x)_i \le 1$. Since the previous result holds for $1 \le i \le d$, we conclude $x \in \{x \in \mathbb{R}^d : -1 \le (x)_i \le 1$, for all $1 \le i \le d\}$, so we have completed this part of the proof.

Now lets check $\{x \in \mathbb{R}^d : -1 \leq (x)_i \leq 1, \text{ for all } 1 \leq i \leq d\} \subseteq \operatorname{convex}\{+1, -1\}^d$:

Apply induction on d. The base case d = 1 is obvious, since $[-1, +1] = convex\{+1, -1\}$. Assume that the result its true for d - 1. Let $v_1, v_2, ..., v_{2^{d-1}}$ be any order of the points in the set $\{+1, -1\}^{d-1}$, now define $u_1 = (v_1, -1), u_2 = (v_2, -1), ..., u_{2^{d-1}} = (v_{2^{d-1}}, -1)$, and $w_1 = (v_1, +1), w_2 = (v_2, +1), ..., w_{2^{d-1}} = (v_{2^{d-1}}, +1)$, so $u_1, u_2, ..., u_{2^{d-1}}, w_1, w_2..., w_{2^{d-1}}$ are all the points of the set $\{+1, -1\}^d$. Let $x \in \{x \in \mathbb{R}^d : -1 \leq (x)_i \leq 1, \text{ for all } 1 \leq i \leq d\}$, then $x = (\hat{x}, x_d)$, where $\hat{x} \in \{x \in \mathbb{R}^{d-1} : -1 \leq (x)_i \leq 1\}$, and $x_d \in [-1, +1]$. By induction hypothesis $\hat{x} \in convex\{+1, -1\}^{d-1}$ so $\hat{x} = \lambda_1 v_1 + \lambda_2 v_2 \dots + \lambda_{2^{d-1}} v_{2^{d-1}}$, a convex combination. Since $x_d \in [-1, +1]$, we know that $x_d = -1\mu + 1\lambda$ a convex combination. Now observe that $(\lambda_k \mu)u_k + (\lambda_k \lambda)w_k = \lambda_k(\mu(v_k, -1) + \lambda(v_k, +1)) = \lambda_k(v_k, x_d)$, for $1 \leq k \leq 2^{d-1}$, so we get that:

$$\begin{aligned} &(\lambda_1 \mu) u_1 + (\lambda_1 \lambda) w_1 + (\lambda_2 \mu) u_2 + (\lambda_2 \lambda) w_2 + \ldots + (\lambda_{2^{d-1}} \mu) u_{2^{d-1}} + (\lambda_{2^{d-1}} \lambda) w_{2^{d-1}} \\ &= \lambda_1 (v_1, x_d) + \lambda_2 (v_2, x_d) + \ldots + \lambda_{2^{d-1}} (v_{2^{d-1}}, x_d) \\ &= (\lambda_1 v_1 + \lambda_2 v_2 \ldots + \lambda_{2^{d-1}} v_{2^{d-1}}, (\lambda_1 + \lambda_2 + \ldots + \lambda_{2^{d-1}}) x_d) \\ &= (\hat{x}, x_d) = x. \end{aligned}$$

Since $(\lambda_1\mu) + (\lambda_1\lambda) + (\lambda_2\mu) + (\lambda_2\lambda) + \dots + (\lambda_{2^{d-1}}\mu) + (\lambda_{2^{d-1}}\lambda) = (\mu+\lambda)(\lambda_1+\lambda_2+\dots+\lambda_{2^{d-1}}) = 1$, and all of them are nonnegative, we conclude that $x \in \text{convex}\{+1, -1\}^d$. This complete the proof.