3)Lets prove that convex $\{+1,-1\}^{d}=\left\{x \in R^{d}:-1 \leq x_{i} \leq 1\right.$, for all $\left.1 \leq i \leq d\right\}$ :

I will check first convex $\{+1,-1\}^{d} \subseteq\left\{x \in R^{d}:-1 \leq(x)_{i} \leq 1\right.$, for all $\left.1 \leq i \leq d\right\}$ : Let $v_{1}, v_{2}, \ldots, v_{2^{d}}$ be any order of the points in the set $\{+1,-1\}^{d}$. Let $x \in \operatorname{convex}\{+1,-1\}^{d}$, so we
write $x=\lambda_{1} v_{1}+\lambda_{2} v_{2}+\ldots+\lambda_{2^{d}} v_{2^{d}}$, where $\lambda_{1}+\lambda_{2}+\ldots+\lambda_{2^{d}}=1$, and $\lambda_{j} \geq 0$ for all $j$. Observe that $(x)_{i}=\left(\lambda_{1} v_{1}+\lambda_{2} v_{2}+\ldots+\lambda_{2^{d}} v_{2^{d}}\right)_{i}=\lambda_{1}\left(v_{1}\right)_{i}+\lambda_{2}\left(v_{2}\right)_{i}+\ldots+\lambda_{2^{d}}\left(v_{2^{d}}\right)_{i}$; we know that $\left(v_{k}\right)_{i} \in\{+1,-1\}$ for $1 \leq k \leq 2^{d}$, so its also true that $-1 \leq\left(v_{k}\right)_{i} \leq 1$. Since $\lambda_{k} \geq 0$, using the previous inequality, we get $-\lambda_{k} \leq \lambda_{k}\left(v_{k}\right)_{i} \leq \lambda_{k}$, for all $k$. Addindg this inequalities we get $-\left(\lambda_{1}+\lambda_{2}+\ldots+\lambda_{2^{d}}\right) \leq(x)_{i} \leq\left(\lambda_{1}+\lambda_{2}+\ldots+\lambda_{2^{d}}\right) \Rightarrow-1 \leq(x)_{i} \leq 1$. Since the previous result holds for $1 \leq i \leq d$, we conclude $x \in\left\{x \in R^{d}:-1 \leq(x)_{i} \leq 1\right.$, for all $\left.1 \leq i \leq d\right\}$, so we have completed this part of the proof.

Now lets check $\left\{x \in R^{d}:-1 \leq(x)_{i} \leq 1\right.$, for all $\left.1 \leq i \leq d\right\} \subseteq \operatorname{convex}\{+1,-1\}^{d}$ :
Apply induction on $d$. The base case $d=1$ is obvious, since $[-1,+1]=$ convex $\{+1,-1\}$. Assume that the result its true for $d-1$. Let $v_{1}, v_{2}, \ldots, v_{2^{d-1}}$ be any order of the points in the set $\{+1,-1\}^{d-1}$, now define $u_{1}=\left(v_{1},-1\right), u_{2}=\left(v_{2},-1\right), \ldots, u_{2^{d-1}}=\left(v_{2^{d-1}},-1\right)$, and $w_{1}=$ $\left(v_{1},+1\right), w_{2}=\left(v_{2},+1\right), \ldots, w_{2^{d-1}}=\left(v_{2^{d-1}},+1\right)$, so $u_{1}, u_{2}, \ldots, u_{2^{d-1}}, w_{1}, w_{2} \ldots, w_{2^{d-1}}$ are all the points of the set $\{+1,-1\}^{d}$. Let $x \in\left\{x \in R^{d}:-1 \leq(x)_{i} \leq 1\right.$, for all $\left.1 \leq i \leq d\right\}$, then $x=\left(\hat{x}, x_{d}\right)$, where $\hat{x} \in\left\{x \in R^{d-1}:-1 \leq(x)_{i} \leq 1\right\}$, and $x_{d} \in[-1,+1]$. By induction hypothesis $\hat{x} \in$ convex $\{+1,-1\}^{d-1}$ so $\hat{x}=\lambda_{1} v_{1}+\lambda_{2} v_{2} \ldots+\lambda_{2^{d-1}} v_{2^{d-1}}$, a convex combination. Since $x_{d} \in[-1,+1]$, we know that $x_{d}=-1 \mu+1 \lambda$ a convex combination. Now observe that $\left(\lambda_{k} \mu\right) u_{k}+\left(\lambda_{k} \lambda\right) w_{k}=$ $\lambda_{k}\left(\mu\left(v_{k},-1\right)+\lambda\left(v_{k},+1\right)\right)=\lambda_{k}\left(v_{k}, x_{d}\right)$, for $1 \leq k \leq 2^{d-1}$, so we get that:
$\left(\lambda_{1} \mu\right) u_{1}+\left(\lambda_{1} \lambda\right) w_{1}+\left(\lambda_{2} \mu\right) u_{2}+\left(\lambda_{2} \lambda\right) w_{2}+\ldots+\left(\lambda_{2^{d-1}} \mu\right) u_{2^{d-1}}+\left(\lambda_{2^{d-1}} \lambda\right) w_{2^{d-1}}$
$=\lambda_{1}\left(v_{1}, x_{d}\right)+\lambda_{2}\left(v_{2}, x_{d}\right)+\ldots+\lambda_{2^{d-1}}\left(v_{2^{d-1}}, x_{d}\right)$
$=\left(\lambda_{1} v_{1}+\lambda_{2} v_{2} \ldots+\lambda_{2^{d-1}} v_{2^{d-1}},\left(\lambda_{1}+\lambda_{2}+\ldots+\lambda_{2^{d-1}}\right) x_{d}\right)$
$=\left(\hat{x}, x_{d}\right)=x$.
Since $\left(\lambda_{1} \mu\right)+\left(\lambda_{1} \lambda\right)+\left(\lambda_{2} \mu\right)+\left(\lambda_{2} \lambda\right)+\ldots+\left(\lambda_{2^{d-1}} \mu\right)+\left(\lambda_{2^{d-1}} \lambda\right)=(\mu+\lambda)\left(\lambda_{1}+\lambda_{2}+\ldots+\lambda_{2^{d-1}}\right)=1$, and all of them are nonnegative, we conclude that $x \in \operatorname{convex}\{+1,-1\}^{d}$. This complete the proof.

