b) Using the previous result, we just need to count the cardinality of $J^{\circ}(\mathrm{Int}(P))$. For $P = A_n$ we have $\mathrm{Int}(A_n) = A_n$ and as any subset of A_n is a downset, then $|J^{\circ}(\mathrm{Int}(P))| = 2^n$.

For $P = C_n$ we show the answer is the Catalan number C_{n+1}. As shown in Figure 1, $\mathrm{Int}(P)$ looks like a pyramid.

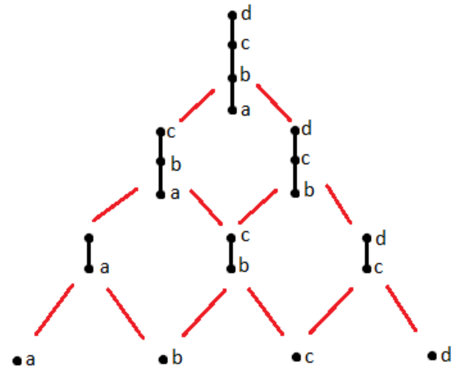

Figure 1: Poset of intervals of C_3

There is a natural bijection from downsets in $\mathrm{Int}(C_n)$ to the Dyck paths of lenght $2(n+1)$. We first add two more rows to the pyramid, and we assume all these new points are also in the downset. We consider the region below the points in the downset. The boundary of this region is a Dyck path.

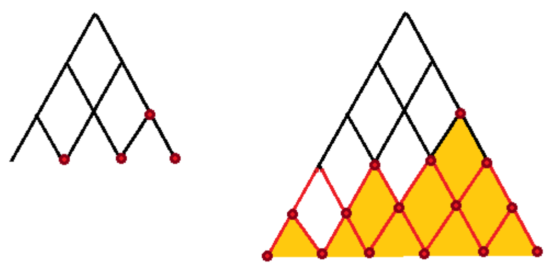

Figure 2: Bijection from downsets in $\mathrm{Int}(C_n)$ to Dyck paths. Left: A downset (points in red) of the poset. Right: The corresponding Dyck path