5. **(a)For a finite poset P regard the set $Int(P) = \{[x,y] : x \leq y$ in $P\}$ as a poset ordered by containment. Let $J^{\circ}(Int(P))$ be the poset of downsets of $Int(P)$ ordered by reverse containment. Prove that the poset of ideals of the incidence algebra is isomorphic to $J^{\circ}(Int(P))$.**

   (I worked with Fernando Torres and Sebastián Osorio)
   Let I be an ideal of the incidence algebra and let's consider $I^{\perp} = \{[x,y] : f([x,y]) = 0$ for all $f \in I\}$. We'll see that $I^{\perp}$ is a downset:

   Let $[x,y] \in I^{\perp}$. If $f \in I$ and $g \in I(P)$ then $gf \in I$ i.e. $gf([x,y]) = 0$ which means:

$$\sum_{x \leq z \leq y} g[x,z]f[z,y] = 0$$

and this works for all $g \in I(P)$. In particular if we take $g = [x,z]^*$ we get $f([z,y]) = 0$ for all $f \in I$, then $[x,z] \in I^{\perp}$ for all $x \leq z \leq y$.

Analogously (since I is a two side ideal) we have $[z,y] \in I^{\perp}$ for all $x \leq z \leq y$ therefore if $[a,b] \subset [x,y]$ we have:

$[x,y] \in I^{\perp} \Rightarrow [x,b] \in I^{\perp} \Rightarrow [a,b] \in I^{\perp}$ Hence $I^{\perp}$ is a downset i.e. $I^{\perp} \in J^{o}(Int(P))$

On the other hand, a downset is clearly a subcoalgebra. Therefore the map $I \to I^{\perp}$ is a biyection.

Finally, let $I \subset J$ be two ideals of $I(P)$. If $f \in J^{\perp}$, then $f([x,y]) = 0$ for all $[x,y] \in J$ then off course $f \in I^{\perp}$. Hence our map preserves the order and our posets are isomorphic.