3) Let (C, Δ, ε) be a coalgebra

$\rho : C^* \otimes C^* \longrightarrow (C \otimes C)^*$ is the canonical isomorphism

hence, $\rho^* : C \otimes C \to (C^* \otimes C^*)^*$ is an isomorphism,

$$\langle m(m \otimes id)x, c \rangle = \langle \Delta^* \rho (m \otimes id) x, c \rangle = \langle (m \otimes id) x, \rho^* \Delta c \rangle$$
$$= \langle x, (m^* \otimes id^*) \rho^* \Delta c \rangle = \langle x, (\rho^* \Delta \otimes id) \rho^* \Delta c \rangle =$$
$$\langle x, (id^* \otimes \rho^* \Delta) \rho^* \Delta c \rangle = \langle (id \otimes m) x, \rho^* \Delta c \rangle =$$
$$\langle m(id \otimes m) x, c \rangle$$

as $\rho^* \Delta$ is "coassociative" because ρ^* is an isomorphism and Δ is coassociative

now we take $u := \varepsilon^* : \mathbb{K} \longrightarrow C^*$, $r \in \mathbb{K}$, $x \in C^*$

$$\langle m(u \otimes id) r \otimes x, c \rangle = \langle u \otimes id (r \otimes c), \rho^* \Delta^* c \rangle$$
$$= \langle r \otimes x, (u^* \otimes id^*) \rho^* \Delta c \rangle = \langle r \otimes x, (\varepsilon \otimes id^*) \rho^* \Delta c \rangle$$
$$= \langle r \otimes x, I c \rangle = \langle I^*(r \otimes x), c \rangle$$

$I : C \to \mathbb{K} \otimes C$ is the canonical isomorphism hence

$I^* (\mathbb{K} \otimes C)^* \to C^*$ is the canonical isomorphism.

checking the right side of the commutativity of the unit diagram is analogous.