6. $\mathbb{R}[x, y]$ is a finitely generated algebra. Take the subalgebra generated by $\{xy^i : i \geq 1\}$. Suppose it is finitely generated with set of generators $\{a_k\}$. take $r = \max_k \deg_y(a_k)$, $m = \max_k \deg_x(a_k)$. Notice that for any monomial v of this subalgebra $\deg_x(v) \leq \deg_y(v)$. Take n an integer greater than mr and divisible by m. It is easy to see that $x^m y^n$ is an element of this subalgebra. To achieve the degree of y in $x^m y^n$ we would need to raise some generator to some power greater than m but then we would get some element with $\deg_x > m$ so it can't be $x^m y^n$. This is a contradiction and hence this subalgebra is not finitely generated.