# 1. Exercise 1.

1.1. **Dull sequences must be the same as set partitions.** Let $\mathcal{A}(m+n,m)$ denote the set of dull sequences of length $n+m \geq 1$ and maximum $m \geq 1$ and let $A(n+m,m)$ be its cardinality. Define the combinatorial class $\mathcal{A}(m+\cdot,m)$ that contains the sequences in $\mathcal{A}(m+n,m)$ for any $n \geq 0$ along with the size function $|\cdot| =$"length of the sequence".

Thanks to the definition of dull sequences it is possible to split any element in the class $\mathcal{A}(m+\cdot,m)$ into some "irreducible" dull sub-sequences as follows:

$$\overbrace{1a_{11}...a_{1\alpha_1}}^{A_1}\ \overbrace{2a_{21}...a_{2\alpha_2}}^{A_2}\ ...\ \overbrace{ma_{m1}...a_{m\alpha_m}}^{A_m}$$

where each of the sub-sequences $A_i$ only has elements in $[i]$ and its first element is $i$. If we define the sub class $\mathcal{A}_i$ as the set of all sequences $A_i$ that satisfy the above property, then we clearly have that

$$\mathcal{A}_i(z) = \sum_{n=1}^{\infty} i^{n-1}z^n = \frac{z}{1-iz},$$

since for each $A_i = ia_{1i}...a_{1n}$ the position $a_{ij}$ can be any element of $[i]$. Hence, we have that

$$\begin{aligned}\mathcal{A}(m+\cdot,m)(z) &= \mathcal{A}_1(z)\cdot\mathcal{A}_2(z)\cdot\ldots\cdot\mathcal{A}_m(z)\\ &= \frac{z}{1-z}\frac{z}{1-2z}\cdots\frac{z}{1-mz},\end{aligned}$$

but in Lecture 12 we saw that the RHS of the above equation was the GF of the Stirling Numbers, and hence we must have that $A(m+n,m) = S(n+m,m)$.