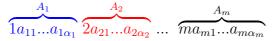
1. EXERCISE 1.

1.1. Dull sequences must be the same as set partitions. Let $\mathcal{A}(m+n,m)$ denote the set of dull sequences of length $n+m \geq 1$ and maximum $m \geq 1$ and let $\mathcal{A}(n+m,m)$ be its cardinality. Define the combinatorial class $\mathcal{A}(m+\cdot,m)$ that contains the sequences in $\mathcal{A}(m+n,m)$ for any $n \geq 0$ along with the size function $|\cdot| =$ "length of the sequence".

Thanks to the definition of dull sequences it is possible to split any element in the class $\mathcal{A}(m + \cdot, m)$ into some "irreducible" dull sub-sequences as follows:



where each of the sub-sequences A_i only has elements in [i] and its first element is i. If we define the sub class A_i as the set of all sequences A_i that satisfy the above property, then we clearly have that

$$\mathcal{A}_i(z) = \sum_{n=1}^{\infty} i^{n-1} z^n = \frac{z}{1-iz},$$

since for each $A_i = ia_{1i}...a_{1n}$ the position a_{ij} can be any element of [i]. Hence, we have that

$$\mathcal{A}(m+\cdot,m)(z) = \mathcal{A}_1(z) \cdot \mathcal{A}_2(z) \cdot \dots \cdot \mathcal{A}_m(z)$$
$$= \frac{z}{1-z} \frac{z}{1-2z} \dots \frac{z}{1-mz},$$

but in Lecture 12 we saw that the RHS of the above equation was the GF of the Stirling Numbers, and hence we must have that A(m+n,m) = S(n+m,m).