4. • Let x, y, q be elements of a non-commutative ring such that $yx = qxy$, and q commutes with x and y.Consider $(x+y)^n$, let $i \in [n]$ lets find the coefficient A of $x^i y^{n-i}$. Consider the multiset $M = \{x^i y^{n-i}\}$, we

know that in order to get how many times appears $x^i y^{n-i}$ we have to consider all the possible permutations of M, and, by knowing that q commutes with xy, reorder each permutation to get $x^i y^{n-i}$. Let $\pi \in S_M$, suppose that $x < y$ so that every time that y appears before an x it will be recorded as an inversion of π. Because q commutes with yx, whenever we set $yx = qxy$ we will get a q. Hence q will appear $inv(\pi)$ times, that is the coefficient of each $x^i y^{n-i}$ is set by:

$$\text{A} = \sum_{\pi \in S_M} q^{inv(\pi)} = \binom{n}{i, n-i}_q = \binom{n}{i}_q$$

Since this happens for each i we have:

$$(x+y)^n = \sum_{i=0}^{n} \binom{n}{i}_q x^i y^{n-i}$$

Which is what we wanted to prove.

- Let $x_1, ., ., x_m, q$ be elements of a non-commutative ring such that $x_j x_i = q x_i x_j$ for $i < j$ and $x_i q = q x_i$ for all i. Consider $(x_1 + ... + x_m)^n$, let $\alpha_1, , , \alpha_m$ be a m-composition of n lets find the coefficient A of ${x_1}^{\alpha_1} ... {x_m}^{\alpha_m}$. Consider the multiset$M = \{{x_1}^{\alpha_1} ... {x_m}^{\alpha_m}\}$, we know that in order to get how many times appears ${x_1}^{\alpha_1} ... {x_m}^{\alpha_m}$ we have to consider all the permutations of M and, by knowing that q commutes as in the hypothesis, reorder each permutation to get ${x_1}^{\alpha_1} ... {x_m}^{\alpha_m}$. Let $\pi \in S_M$, suppose that $x_i < x_j$ for all $i < j$ so that every time that x_j appears before an x_i it will be recorded as an inversion of π. Because q commutes as stated in the hypothesis, whenever we set $x_j x_i = q x_i x_j$ for $i < j$ we will get a q. Hence q will appear $inv(\pi)$ is set by:

$$\text{A} = \sum_{\pi \in S_M} q^{inv(\pi)} = \binom{n}{\alpha_1, , , \alpha_m}_q$$

Since this happens for each m-composition of n we have

$$(x_1 + ... + x_m)^n = \sum_{\alpha} {}_1 + ... + \alpha_m = n \binom{n}{\alpha_1, , , \alpha_m}_q {x_1}^{\alpha_1} ... {x_m}^{\alpha_m}$$

Which is what we wanted to prove.

(I worked with Nicolas Peña)