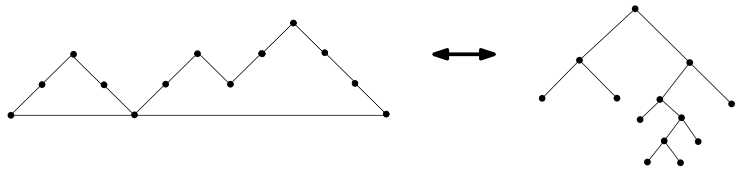
3. Consider the following bijection $f$ between Dyck Paths and binary trees. Let $P$ be a Dyck of length $2n$. Construct the following binary tree. Start in $(0,0)$ and draw the root of $f(P)$. Each time you go up in $P$, draw the left son from the last vertex you drew in $f(P)$. Each time you go down in $P$, go up one vertex in $f(P)$ and draw the right son of that vertex. The figure shows an example.



This is a bijection as you can go back. Let $T$ be a binary tree. Start in the root of $T$. If you can go down left in $T$, do it and go up in your Dyck Path. If you can't, then go up until you can go down right where you haven't been before in $T$ and go down in the Dyck Path. By construction this is clearly $f^{-1}$, so it is in fact a bijection.

Now if you go up $a(P)$ steps before going down, when you draw $f(P)$ you draw $a(P)$ left sons. Then, when you go down for the first time in $P$, you go up in $f(P)$ and you never go back to the leftmost branch in $f(P)$. So $a(P)$ is the length (in edges) of the left most branch. Now if at a given step you touch back the floor in $P$, when drawing $f(P)$ this means you have drawn as many right sons as left sons. The last right son you drew must be in the right most branch as you are drawing the tree from left to right. So each time $P$ touches the floor is an edge in the rightmost branch of $f(p)$.

Now if $T$ is a binary tree let $T'$ be the binary tree resulting from reflecting horizantally $T$. If $P$ is a Dyck path let $P'$ be $f-1(f(P)')$. Cleary $T'' = T$ so $P'' = P$. So $f(P)$ has the leftmost branch of length $a(P)$ and rightmost branch of length $b(P)$. Then $f(P)'$ has the leftmost branch of length $b(P)$ and rightmost branch of length $a(P)$. Then $a(P') = b(P)$ and $b(P') =$

$a(P)$. As $'$ is a bijection of Dyck Paths on themselves, so

$$\begin{aligned}
\sum_{P Dyck} x^{a(P)} y^{b(P)} &= \sum_{P Dyck} x^{a(P')} y^{b(P')} \\
&= \sum_{P Dyck} x^{b(P)} y^{a(P)}
\end{aligned}$$

(worked with Felipe Suarez)