2. If a permutation $w = w_1 \dots w_n$ avoid the pattern abc then the reverse permutation $w_n \dots w_1$ avoid the pattern cba. Since in some lecture is proved that the number of permutations in S_n avoiding 321 is C_n then the permutations avoiding 123 is C_n.

For 231: Let proceed by induction, the case for $n = 1, 2, 3$ is clear, so assume that the number of permutation of S_t, $t < n$, which avoids 312 is C_t. Let $S_n(k)$ the permutations $w = w_1 \dots w_n \in S_n$ such that $w_k = n$, then :

$$w \text{ avoids } 231 \Leftrightarrow w_1 \dots w_k \text{ and } w_{k+1} \dots w_n \text{ avoids } 231$$

($\Rightarrow$) This direction is evident. For $\Leftarrow$, by contradiction suppose that w has 231 pattern, and let $w_a w_b w_c$, $a < b < c$, be the pattern in w. If $c \leq k$ or $a > k$ there is a contradiction because $w_a w_b w_c$ would be in $w_1 \dots w_k$ or in $w_k \dots w_n$, then $a < k$ and $c > k$, but this is a contradiction too because $w_a w_b 1$ is in $w_1 \dots w_k$ and has the pattern 231. Since is possible to identify $w_1 \dots w_k$ and $w_{k+1} \dots w_n$ with some permutations of S_k and S_{n-1-k} that avoids 231, so $\#S_n(k) = C_k C_{n-1-k}$ by the induction hypothesis. Considering all possible values of k we obtain the number of permutations in S_n that avoid 231, then there are $\sum_{k=1}^{n} C_k C_{n-1-k}$, which is the Catalan number C_n