(c) We claim that, if we view x as a positive integer, the left- and right-hand sides of the equation both count the number of ways to choose n balls from a collection of n blue balls and n red balls and to assign a number from $[x]$ to each of the blue balls selected.

On the left-hand side, we condition on k, the number of blue balls selected. Note that if we select k blue balls, we must select $n-k$ red balls. There are $\binom{n}{k}$ ways to select k blue balls from the n blue balls, $\binom{n}{k}$ ways to select $n-k$ red balls from the n red balls (since $\binom{n}{k} = \binom{n}{n-k}$), and x^k ways to assign numbers to the blue balls we've chosen. Hence, there are $\binom{n}{k}^2 x^k$ ways to select n balls, k of which are blue, and to assign a number from $[x]$ to each of the selected blue balls. Summing over k yields the total number of ways to select n balls and assign numbers to the selected blue balls.

On the right-hand side, we condition on j, the number of blue balls selected and assigned numbers in $[x-1]$. Notice that j can be any number between 0 and n. We can select n balls and assign j of the selected blue balls numbers in $[x-1]$ by first selecting j of the n blue balls ($\binom{n}{j}$ ways), assigning each a number in $[x-1]$ ($(x-1)^j$ ways), and then selecting $n-j$ of the remaining $2n-j$ balls and assigning the number x to any blue balls among these $n-j$ balls ($\binom{2n-j}{n}$ ways, since $\binom{2n-j}{n} = \binom{2n-j}{n-j}$). Summing over j thus yields the total number of ways to select n balls and assign numbers in $[x]$ to the selected blue balls.

It follows from the above that the left- and right-hand sides are equal for every $x \in \mathbf{Z}_{>0}$. Since the left- and right-hand sides are both polynomials in x that agree for infinitely many values of x, they must be equal (as polynomials).