For this exercise I follow an idea suggested by Jose Gabriel Acevedo. We count bounded compositions depending on the number of parts in the composition. Since $m_i \leq k-1$, for all $i = 1, \dots, l$, an integer n which admits a k-bounded composition of l parts must be $\leq l(k-1)$, so, all possible k-bounded compositions of all possible n appear, up to the correspondence, in the terms of $(x + x^2 + \cdots + x^{k-1})^l$. Since we need the compositions of all possible n's, we sum over l in order to have all the bounded compositions of any number of parts. Hence, we get that $c_k(n)$ is the n-th coefficient of the formal power series

$$\sum_{n=0}^{\infty} (x + x^2 + \cdots + x^{k-1})^n.$$

Finally, easy algebraic computations lead us to

$$\begin{aligned}
\sum_{j=0}^{\infty} (x + x^2 + \cdots + x^{k-1})^j &= \frac{1}{1 - (x + x^2 + \cdots + x^{k-1})} \\
&= \frac{1}{1 - x(1 + x + \cdots + x^{k-2})} \\
&= \frac{1}{1 - x\frac{1-x^{k-1}}{1-x}} \\
&= \frac{1-x}{(1-x)(1 - \frac{x-x^k}{1-x})} \\
&= \frac{1-x}{1-2x+x^k},
\end{aligned}$$