Another way to represent the compositions of compositions involves dots bars and squiggles.

Draw n dots separated by $n-1$ spaces.

$$\bullet \quad \bullet \quad \bullet \quad \bullet \quad := [4]$$

Placing $k-1$ vertical lines in $k-1$ of the spaces partitions n (represented by n $\bullet$'s) into k parts.

$$\bullet \mid \bullet \quad \bullet \quad \bullet \quad := [1+3]$$

Further partition the partition (or rather, further decompose the composition) by placing a squiggle in any of the remaining spaces.

$$\bullet \mid \bullet \wr \bullet \quad \bullet \quad := [1+(1+2)]$$

$$\bullet \mid \bullet \quad \bullet \wr \bullet \quad := [1+(2+1)]$$

$$\bullet \mid \bullet \wr \bullet \wr \bullet \quad := [1+(1+1+1)]$$

For any n there are $n-1$ spaces. In any space you may draw a bar, a squiggle or nothing at all. All compositions of compositions of n can be schematically represented in this manner. So the number of compositions of compositions of n is:

$$3^{n-1}$$

$n-1$ is the number of spaces

for each space you have 3 options: $\mid$, $\wr$ or blank. $\square$