

(d) Consider the following algorithm: first choose which between B or C is going to be the next consonant (supposing there is one). Then for each slot, if its the first one or its preceeded by an A , choose whether to put an A or to put the next consonant, and if it is preceeded by a

consonant, choose again which is going to be the next consonant, either B or C . In any case place an A in that slot. For each decision we have 2 choices, for a total of 2^{n+1} possibilities. Now we have generated all possible words, but for those words which a next consonant was chosen but never placed (the ones which finished with an A), we are counting them twice. But there are clearly w_{n-1} words ending with an A . So we have $w_n = 2^{n+1} - w_{n-1}$. Applying that identity several times we end up with

$$\begin{aligned}w_k &= \sum_{i=0}^{k+1} 2^{n+1-i} (-1)^i \\ &= \frac{2^{n+2} - (-1)^{n+2}}{2 - (-1)} \\ &= \frac{2^{n+2} - (-1)^n}{3}\end{aligned}$$