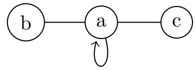
We must show that the number of words of length n using only A, B or C as letters and having no more than two or more consecutive consonants is

$$a_n = \frac{2^{n+2} + (-1)^{n+1}}{3}$$

Solution If you consider the graph



Then the problem is equivalent to counting the number of walks of length $n-1$ in this graph. From basic graph theory, if A is the adjacency matrix of a graph G, then $A^k(i,j)$ counts the number of walks from node i to node j with length k. In this case

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

So if i need to count all walks of size n, I only need to add all entries of A^n. Equivalently,

$$a_{n+1} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} A^n \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

After diagonalizing, I can easily compute A^n,

$$A = \begin{bmatrix} -1 & 0 & 2 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} -1/3 & 1/3 & 1/3 \\ 0 & -1/2 & 1/2 \\ 1/3 & 1/6 & 1/6 \end{bmatrix}$$

$$\begin{aligned} \Rightarrow a_{n+1} &= \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} A^n \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} && (1) \\ &= \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 2 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} (-1)^n & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2^n \end{bmatrix} \begin{bmatrix} -1/3 & 1/3 & 1/3 \\ 0 & -1/2 & 1/2 \\ 1/3 & 1/6 & 1/6 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 4 \end{bmatrix} \begin{bmatrix} (-1)^n & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2^n \end{bmatrix} \begin{bmatrix} 1/3 \\ 0 \\ 2/3 \end{bmatrix} && (2) \\ &= \frac{2^{n+3} + (-1)^n}{3} \end{aligned}$$

As desired :D