1. Numeric exercises

   (a) For this exercise there are three *independent* considerations:

      i. First we must know what the third digit from the code is, and there are 8 possibilities for this.

      ii. Second, since the length of the code is 5 digits, there must be either (3, 1, 1) or (2, 2, 1) instances of the digits respectively, therefore, we have $\binom{5}{3,1,1} = \frac{5!}{3!1!1!} = 20$ or $\binom{5}{2,2,1} = \frac{5!}{2!2!1!} = 30$ possibilities respectively.

      iii. Finally, we must choose which digit corresponds to which role in the previous item. However, since we notice there is one distinct number in *either* of the cases (the ones which have 3 and 1 appearances respectively), to choose this digit we have another 3 possibilities: 1, 7, or our new digit.

   Summing this up (actually by the multiplication principle), our numeric answer is:

$$8 \times \left(\binom{5}{3,1,1} + \binom{5}{2,2,1} \right) \times 3 = 8 \times (20 + 30) \times 3 = 1200$$

   (b) Since we must get from (0,0) to (8,8) in 16 unit steps, we may only traverse in the up ($\uparrow$) and right ($\rightarrow$) directions (assuming we may only walk vertically or horizontally) and, so, we may count the paths by $\binom{n+m}{n}$ where n and m are the number of right and up steps respectively (hence $n+m$ corresponds to the length of the path or number of steps). Now, to get from (0,0) to (2,3) the path starts out with 2 (out of the first 5 steps) going right, hence there are $\binom{5}{2} = 10$ possibilities for this. Following up, the path may continue in $\binom{16-5}{8-2} = \binom{11}{6}$ ways. However, out of these, there are a series of paths which *do* pass through point (6,5), namely $\binom{11-5}{6-2}\binom{5}{2}$ of them (since out of the 6 steps from (2,3) to (6,5) four go right, and out of the 5 from (6,5) to (8,8) 2 go right. Numerically, this is:

$$\binom{5}{2}\left(\binom{11}{6} - \binom{6}{4}\binom{5}{2}\right) = 10(462 - 15 \times 10) = 3120$$

   (c) First of all, since the top and bottom teams are in known/fixed positions, we may omit them from our count. Second, since Tolima is immediately above Santa Fé, when counting they may be treated as a single entity (and then replaced back in the correct order). This gives us $(18-2-1)! = 15!$ possible orders for the teams as a starting point. The other condition states that there are three other teams which are to be included in a specific order, so that the previous number counts 3! each valid order. Therefore, the numeric answer is simply: $\frac{15!}{3!}$. *(The actual number is 217945728000 but the previous answer is more revealing about the count.)*