

So $\begin{bmatrix} 1 & & \\ & -1 & \\ & & \begin{bmatrix} 2x & & x \\ x & 2x & \\ & x & \ddots \\ x & & x & 2 \end{bmatrix} & \\ & & & & 1 \end{bmatrix} > 0$

$x = -2\cos\left(\frac{\pi}{n+1}\right) \leq -2\cos\left(\frac{\pi}{3}\right) = -1$
 sum of entries
 $2n + (2n \text{ things } \leq -1) \leq 0$

3. Γ has at most one edge ≥ 3

If not, take an induced $\text{---} \overset{p}{\bullet} \text{---} \text{---} \overset{q}{\bullet} \text{---}$ $p, q \geq 3$

So $\begin{bmatrix} 2 & -2\cos\frac{\pi}{p} & & & & \\ -2\cos\frac{\pi}{p} & 2 & -1 & & & \\ & -1 & 2 & & & \\ & & & \ddots & & \\ & & & & 2 & -1 \\ 0 & & -1 & 2 & -2\cos\frac{\pi}{q} & \\ & & & -2\cos\frac{\pi}{q} & 2 & \end{bmatrix}$ pos. def

So $2n - 2(n-2) - 4\cos\frac{\pi}{p} - 4\cos\frac{\pi}{q} > 0$
 $1 - \cos\frac{\pi}{p} - \cos\frac{\pi}{q} > 0$
 $\cos\frac{\pi}{p} \geq \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} > \frac{1}{2}$

4. If Γ has an edge ≥ 3 , it is a straight line

If not, have an induced $\text{---} \overset{p}{\bullet} \text{---} \text{---} \text{---} \text{---} \overset{p}{\bullet} \text{---}$

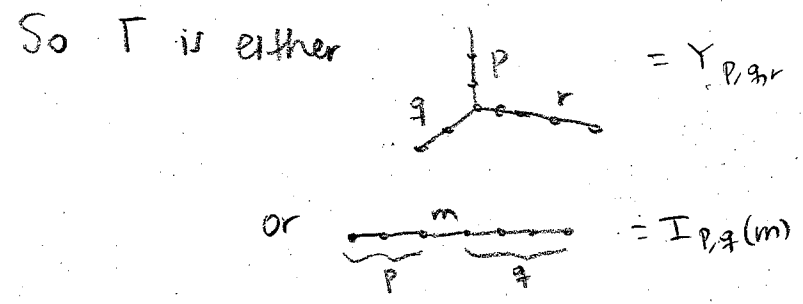
$d(\Gamma) = 2d(D_{n-1}) - 4\cos^2\frac{\pi}{p} d(D_{n-2}) \leq 8 - 16\cos^2\frac{\pi}{p} \leq 0$

5. Γ has at most one branching point
 If not, have an induced $\text{---} \overset{p}{\bullet} \text{---} \text{---} \overset{q}{\bullet} \text{---}$

$d(\Gamma) = 2d(D_{n-1}) - d(D_{n-2} \times A_1) = 2 \cdot 4 - 4 \cdot 2 = 0$

6. Γ has no branching point with ≥ 4 branches

Otherwise, have an induced $\text{---} \overset{p}{\bullet} \text{---} \text{---} \overset{q}{\bullet} \text{---}$ and $d(X) = 2d(X) - d(\bullet) = 2 \cdot 4 - 2 \cdot 2 \cdot 2 = 0$



Check: $d(\text{---} \overset{p}{\bullet} \text{---} \text{---} \text{---} \text{---} \overset{q}{\bullet} \text{---}) = 0 \rightarrow$ if $r \geq 5$ must be $Y_{1,1,r} = D_{r+3}$

$d(\text{---} \overset{p}{\bullet} \text{---}) = 0 \rightarrow$ can't have $p, q, r \geq 2$

So must be $Y_{1,3,3}, Y_{1,2,4}, Y_{1,2,5}$, or $Y_{1,1,r}$ E_6, E_7, E_8, D_{r+3}

Check: $d(\overset{\geq 6}{\bullet})$, $d(\overset{5}{\bullet})$, $d(\overset{4}{\bullet})$, $d(\overset{4}{\bullet}) \leq 0$
 If ≥ 6 then $I_2(m)$ If 5 then H_3 or H_4 If 4 then B_n or F_4

