

Lecture 1
Jan 25, 08

Some unusual things about this course:

◦ sf / bog

- not separate courses!

- work together on e-mail, forum, chat, phone...

◦ final project

- keep thinking about what interests you

◦ syllabus

math.sfsu.edu/federics/coxeter.html

Three ways of thinking about S_3

(and other Coxeter grps)

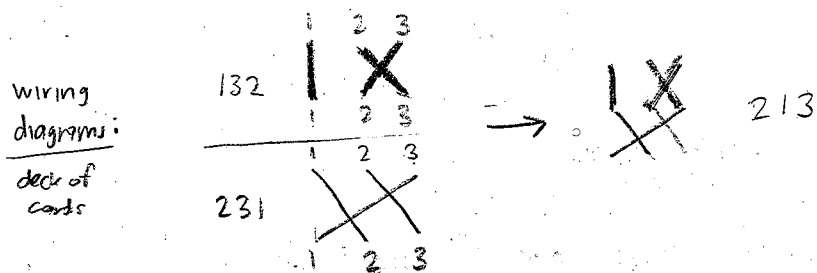
① Combinatorics

elements: perms of $[3] = \{1, 2, 3\}$

$\{123, 132, 213, 231, 312, 321\}$

operation: composition

$$231 \circ 132 = 213$$



② Algebra

generators: a, b

relations: $a^2 = e, b^2 = e, aba = bab$

elements: words on $\{a, b\}$ modulo relations

operation: concatenation "gluing" words

$$\begin{aligned} \text{Ex. } babbababba &= bababab \\ &= abaabab \\ &= b \end{aligned}$$

Can shorten: \circ double letters \rightarrow leave ababa...

$$\circ abab = ba$$

$$\circ baba = ab$$

So elements: $\{e, a, b, ab, ba, aba\}$
"bab

Why the same?

$$a = \times |$$

$$a^2 = \cancel{\times} | = e$$

$$aba = \cancel{\times} \cancel{\times} |$$

$$b = | \times$$

$$b^2 = | \cancel{\times} = e$$

$$bab = | \cancel{\times} \cancel{\times}$$

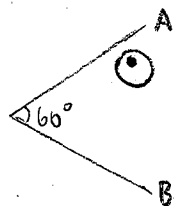
Operation: "stretching wires"

③ Geometry

Kaleidoscope

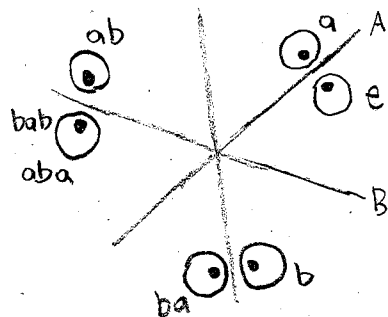


Six reflections



a = reflect on mirror A

b = " " " B



$$a^2 = e$$

$$b^2 = e$$

$$aba = bab$$