Def The <u>Euler char.</u> of a cell cx. X is
$$\chi(X) = \sum_{i=-1}^{d} (-1)^i f_i(X)$$

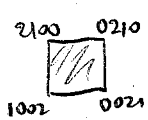
Prop If X is acyclic then $\chi(X) = 0$.

(X irrelevant $\to \chi(X) = -1$)

Pf X acyclic if
$$0 \to \mathbb{F}^{F_d(X)} \to \cdots \to \mathbb{F}^{F_0(X)} \to \mathbb{F}^{F_{-1}(X)} \to 0$$
is exact, and
$$\dim(\mathbb{F}^{F_{-1}(X)}) - \dim(\mathbb{F}^{F_0(X)}) + \cdots \pm \dim(\mathbb{F}^{F_d(X)}) = 0. \qquad \blacksquare$$

Def The <u>$\mathbb{N}^n$-graded Euler char</u> of a labelled cell cx is
$$\chi(X; x_1, \dots, x_n) = \sum_{F \in X} (-1)^{1+\dim F} \mathbf{x}^{a_F}$$

Ex:

$X =$ 

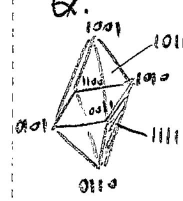
$$\chi(X; a,b,c,d) = 1 - (a^2b + b^2c + c^2d + d^2a) + (a^2b^2c + b^2c^2d + c^2d^2a + d^2a^2b) - a^2b^2c^2d^2$$

Theorem If a labelled cell cx X supports a cellular free resol. of R/I, then
$$H(R/I; \mathbf{x}) = \frac{\chi(X; \mathbf{x})}{(1-x_1)\cdots(1-x_n)}$$

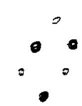
Pf The graded free resol.
$$0 \to R^{F_d(X)} \to \cdots \to R^{F_0(X)} \to R^{F_{-1}(X)} \to R/I \to 0$$
gives
$$H(R/I; \mathbf{x}) = \sum_{i=-1}^{d} (-1)^{i+1} H(R^{F_i(X)}; \mathbf{x})$$
$$= \sum_{i=-1}^{d} (-1)^{i+1} \sum_{\substack{F \in X \\ \dim F = i}} \frac{\mathbf{x}^{a_F}}{(1-x_1)\cdots(1-x_n)} \qquad \blacksquare$$

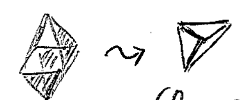
Theorem If a labelled cell cx X supports a cellular free resol. of R/I, then
$$\beta_{i,b}(X) = \dim_{\mathbb{F}} \tilde{H}_{i-1}(X_{<b})$$

Ex.



$$\mathcal{F}_X: 0 \to \underset{1111}{R} \to \underset{\substack{1111\\1111\\1111\\1111\\1110\\1101\\1011\\0111}}{R^8} \to \underset{\substack{1110\\1110\\1110\\1101\\1101\\1101\\1011\\1011\\1011\\0111\\0111\\0111}}{R^{12}} \to \underset{\substack{1100\\1010\\1001\\0110\\0101\\0011}}{R^6} \to \underset{0000}{R} \to R/I \to 0$$

$b = 1110$:  $(\beta_{1,1110} = 2)$

$b = 1111$  $(\beta_{2,1111} = 3)$