Extra: Suppose a is not a zero divisor in a commutative ring R, and M is an R-module. Then the following is an exact sequence and free resolution of $R/\langle a\rangle$.

$$0 \xrightarrow{\delta_2} R \xrightarrow{\delta_1} R \xrightarrow{\delta_0} R/\langle a\rangle \to 0.$$

where δ_1 is multiplication by a.

By tensoring with M we get

$$0 \xrightarrow{\delta_2} R \otimes M \xrightarrow{\delta_1} R \otimes M \xrightarrow{\delta_0} R/\langle a\rangle \otimes M \to 0.$$

But $R \otimes M$ is just M, as any element $r \otimes m$ can be written as $1 \otimes rm$. So we get

$$0 \xrightarrow{\delta_2} M \xrightarrow{\delta_1} M.$$

Then the image of δ_2 in M is just 0. Since δ_1 is multiplication by a, the kernel is $\{m \in M \mid am = 0\}$. Thus the kernel of δ_1 mod the image of δ_2 is $Tor_1(R/\langle a\rangle, M) = \{m \in M \mid am = 0\}$.