3. It suffices to show that $0 \to R/J(-d) \xrightarrow{J} R/I' \xrightarrow{g} R/I \to 0$ is a graded exact sequence, where $d = \deg(m_k)$ and f and g are homomorphisms given by $g : R/J(-d) \to R/I$ so that $[a]_J \mapsto [m_k a]_{I'}$ and $f : R/I' \to R/I$ so that $[a]_{I'} \mapsto [a]_I$. Note that g is degree preserving because of the shifting and f does obviously preserve degree.

   We divide the proof in three steps.

   (a) The sequence is exact in $R/J(-d)$. For this it is enough to prove that $ker(g) = \{0_J\}$ (which in this case is J). Suppose that $g([a]_J) = I'$, this means that $m_k a + I' = I'$, so $m_k a \in I'$, hence every monomial of $m_k a$ is divisible bye some element in I'. It follows that every monomial in a is divisible by (m_k, m_j) for some $j \in \{1, \ldots, n-1\}$. We conclude that a in J, so $[a] = 0_J$.

   (b) The sequence is exact in R/I'. It suffices to show that $Im(g) = Ker(f)$. Let $[a]_{I'} \in Ker(f)$, where a does not have monomials that are multiples of m_j for $j \in \{1, \ldots, k-1\}$. We have that $a + I = I$ so that $a \in I$ so every monomial of a is a multiple of m_k, hence $a \in Im(g)$. Now if $[a]_{I'} \in Im(g)$ (taking the repesentative a of the class with no monomials in I') then there is a polynomial a' such that $a = m_k a'$ and hence $f([a]_{I'}) = f([m_k a']_{I'}) = m_k a' + I = I$ so $[a]_{I'} \in Ker(f)$. We conclude then that $Im(g) = Ker(f)$.

   (c) The sequence is exact in R/I which in this case is equivalent to prove that f is surjective. This follows from the fact that if $[a]_I \in R/I$ then $f([a]_{I'}) = [a]_I$ by definition.

   From (a), (b) and (c) we conclude that $0 \to R/J(-d) \xrightarrow{f} R/I' \xrightarrow{g} R/I \to 0$ is a fininte graded free resolution of R/I (because all the modules are free and the sequence is exact) and so $H_{R/I}(m) = H_{R/I'}(m) - H_{R/J(-d)}(m)$ or equivalently $H_{R/I}(m) = H_{R/I'}(m) - H_{R/J}(m-d)$ as desired.