4.1. Let $a = x_1^{\alpha_1} x_2^{\alpha_2} \ldots x_n^{\alpha_n}$, let $b = x_1^{\beta_1} x_2^{\beta_2} \ldots x_n^{\beta_n}$, and let $a < b$ in the weight order using the above vectors. Since the dot product of the v_is with a and b extracts the ith component of the vector, and since the ith components of a and b are α_i and β_i, respectively, then we know that if $a > b$, there exists some t such that $\alpha_i = \beta_i$ for all $i < t$ and $\alpha_t > \beta_t$. However, this is the same criteria used in the lexicographic order with $x_1 > x_2 > \ldots > x_n$, and so these orderings are equivalent.

4.2. Let $a = x_1^{\alpha_1} \ldots x_n^{\beta_1}$, and let $b = x_1^{\beta_1} \ldots x_n^{\beta_n}$. Assume $a < b$ in the weight order using the above vectors. If deg $a <$ deg b. If $v_1 \cdot multideg(a) < v_1 \cdot multideg(b)$, or $\sum_{k=1}^{n} \alpha_k < \sum_{k=1}^{n} \beta_k$, then deg $a <$ deg b. So far, this weight order coincides with the grevlex order: importance is first given to the degree of the polynomial.

If $v_2 \cdot multideg(a) < v_2 \cdot multideg(b)$, then $\sum_{k=1}^{n-1} \alpha_k + \alpha_n(-n+1) < \sum_{k=1}^{n-1} \beta_k + \beta_+ n(-n+1)$. Here, β_n must be less α_n, and so $\sum_{k=1}^{n-1} \alpha_k < \sum_{k=1}^{n-1} \beta_k$. Again, thus far, this weight order coincides with the grevlex order–the monomial with the larger degree on the last variable is weighted to be the smaller variable.

Assume inductively now that there exists some i such that for all $j > i$, $v_j \cdot multideg(a) = v_j \cdot multideg(b)$, implying (inductively) that $\alpha_j = \beta_j$ for $j > i$. If $\sum_{k=1}^{n-1+i} \alpha_k + \alpha_{n-i+2}(-n + i - 1) < \sum_{k=1}^{n-1} \beta_k + \beta_+ n(-n + i - 1)i$, then we have that $\alpha_{n-i+2} > \beta_{n-i+2}$. Therefore, we have all our conditions for the grevlex order in this weight order: if deg $a <$ deg b, $a < b$. Otherwise, examine the degree of each variable starting with x_n. If $a < b$, then there exists an i such that $\alpha_j = \beta_j$ for all $j > i$ and $\alpha_i > \beta_i$.