2. • Does the *grlex* order have the finite interval property?

  Yes. Between the monomials $a < b$ there are finite total-degrees (the sum of the degrees of each x_i in the monomial) to check, each of them having a finite number of monomials with that total-degree (in fact, the numbers of monomials in n variables with total-degree equal to d is exactly $\binom{d+n-1}{n-1}$). So there exist finitely many monomials c such that $a < c < b$.

  • Does every monomial ordering have the finite interval property?

  No. For *lex* we have $1 < x^k < y$ for infinitely many k.