1. Let R be the ring of continuous functions on $[0,1]$. Let I be the ideal of function that are zero on some neighborhood of zero,

$$I = \{f \in R : \exists\epsilon > 0 \text{ with } f = 0 \text{ on } [0,\epsilon]\}.$$

Let $\langle f_1,\ldots,f_m\rangle$ be any finite subset of I and let $\epsilon_1,\ldots,\epsilon_m > 0$ so that $f_i = 0$ on $[0,\epsilon_i]$. For every $h \in \langle f_1,\ldots,f_m\rangle$, $h = 0$ on $[0,\min\{\epsilon_1,\ldots,\epsilon_m\}]$. Let $0 < \epsilon < \min\{\epsilon_1,\ldots,\epsilon_m\}$ and find $g \in R$ so that $\{g = 0\} = [0,\epsilon]$. Then $g \in I \backslash \langle f_1,\ldots,f_m\rangle$. Thus I cannot be finitely generated.