Welcome!

Where are you joining us from?

Please write it in the Chat box.

Grupo Bahía Trío – Bambuco Viejo

ROBOTS, GEOMETRY, SOCIETY

African Women in Mathematics Seminar
October 5, 2020
The Internet

Federico Ardila Mantilla

San Francisco State University
Universidad de Los Andes
@federicoardila

Thank you for the invitation!

It is a huge honor to be here.

This project is from 2011-2020, with:

Arlys Asprilla, César Ceballos, Hanner Bastidas, John Guo, Matthew Bland
Maxime Pouokam, Megan Owen, Rika Yatchak, Seth Sullivant, Tia Baker

(Cameroon, Colombia, USA)

ROBOTS

When I say “robot”, what robot do you imagine?

Please write it in the Chat box.

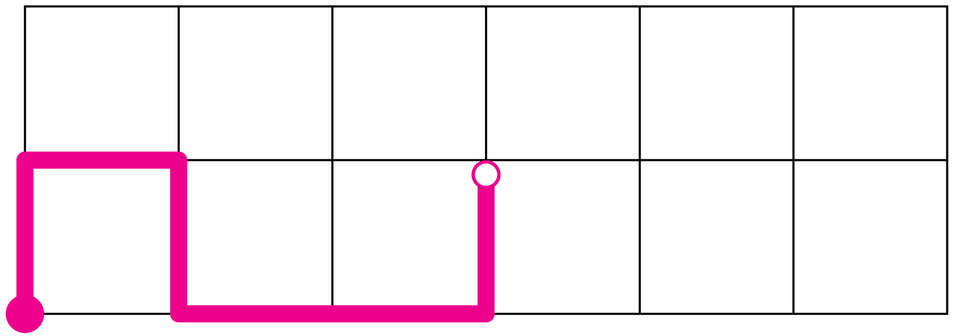
Ford, 1910

Tesla, 2016

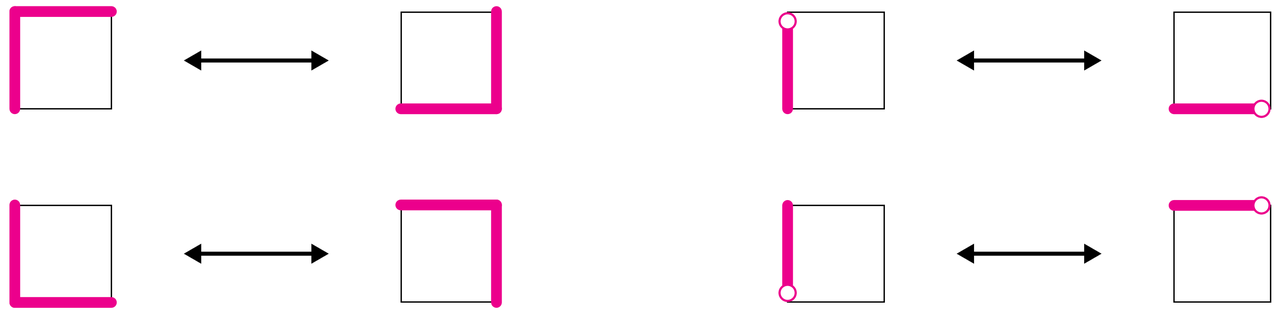
1. ROBOTS

A mathematical model

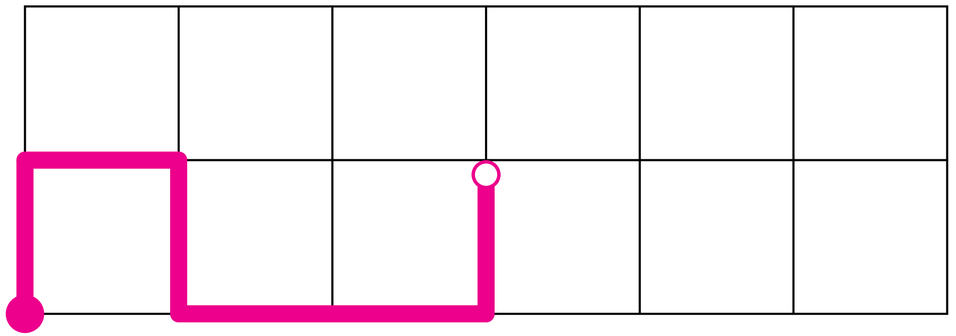
A robotic arm in a tunnel



Allowed movements:



A robotic arm in a tunnel



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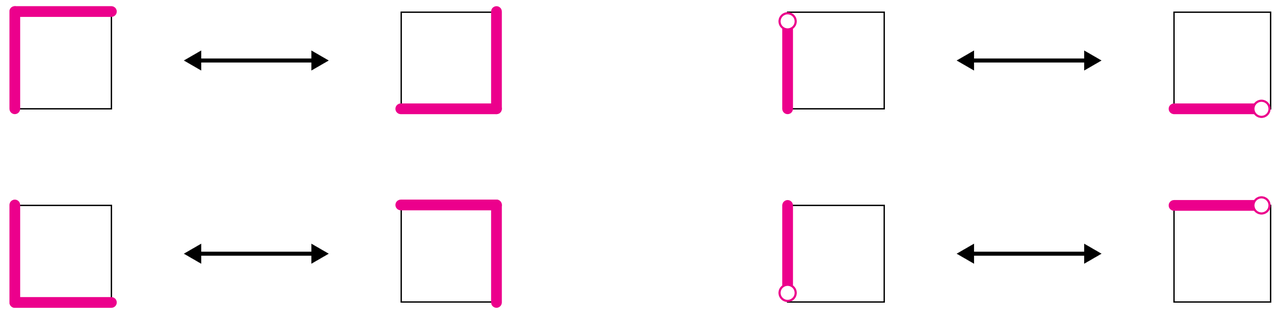
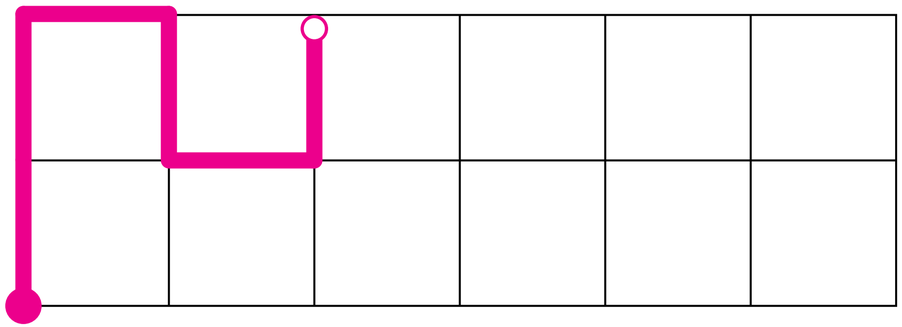
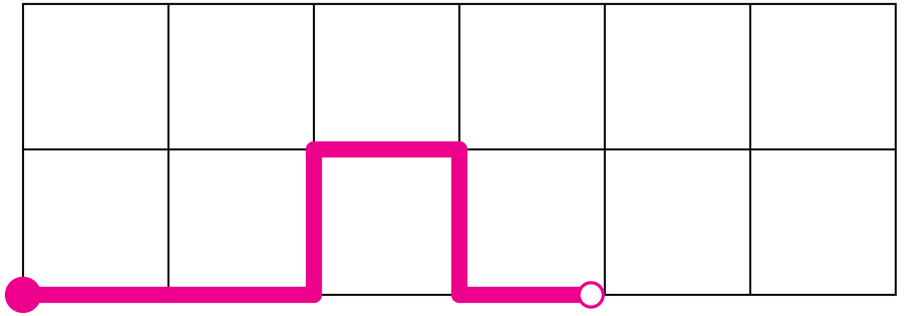


How do we move the robotic arm from one position to another position?





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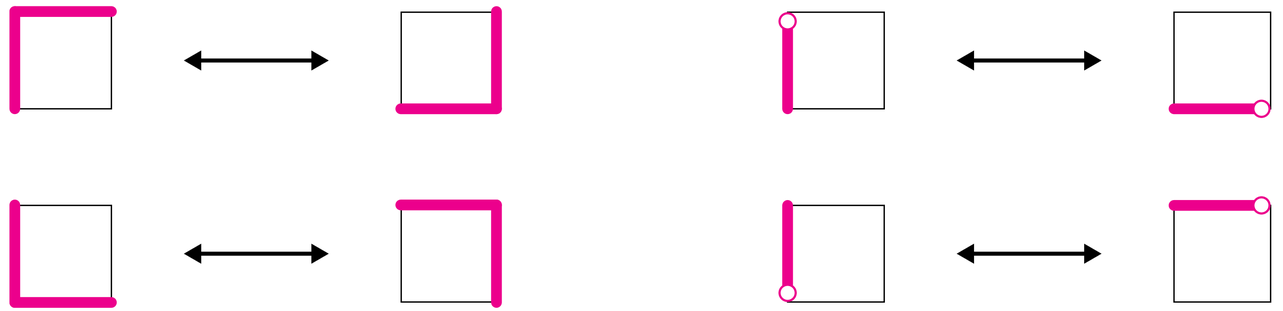
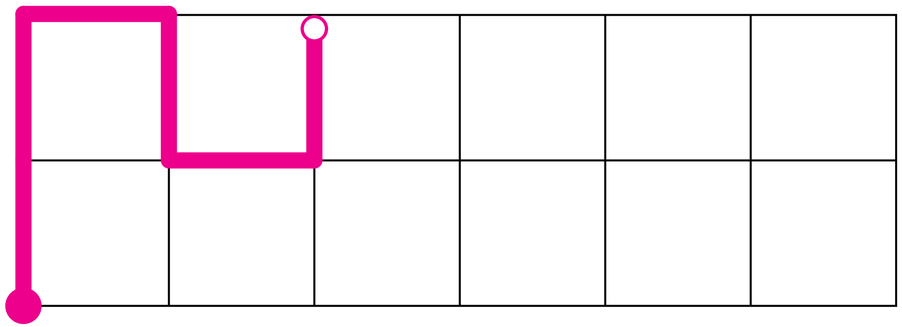
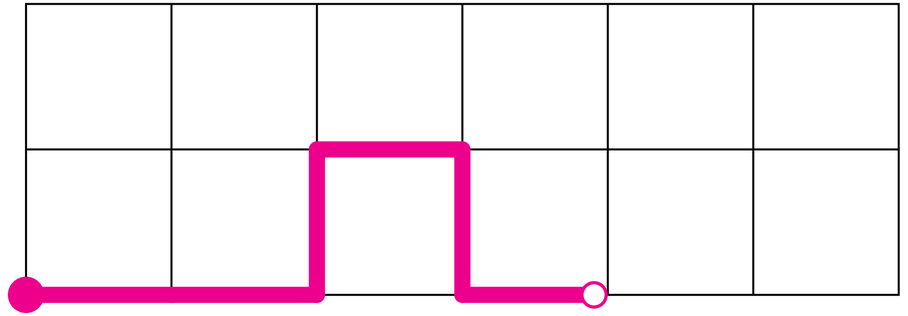


How do we move the robotic arm **optimally** from one position to another position?





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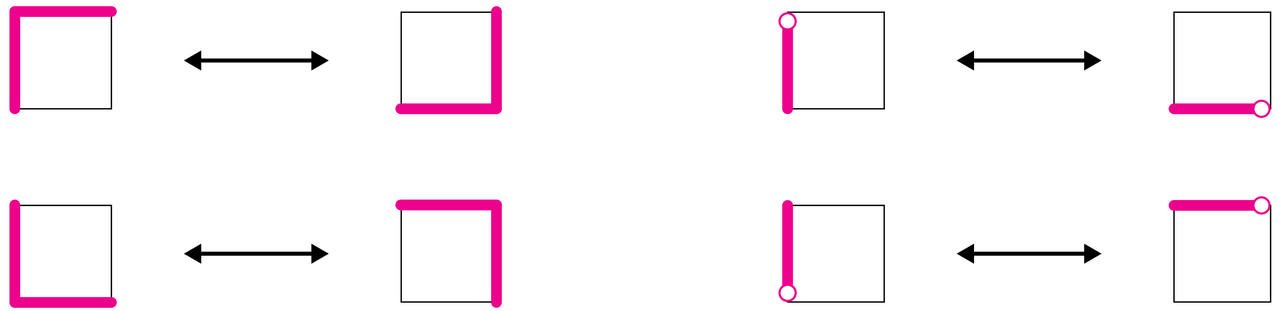
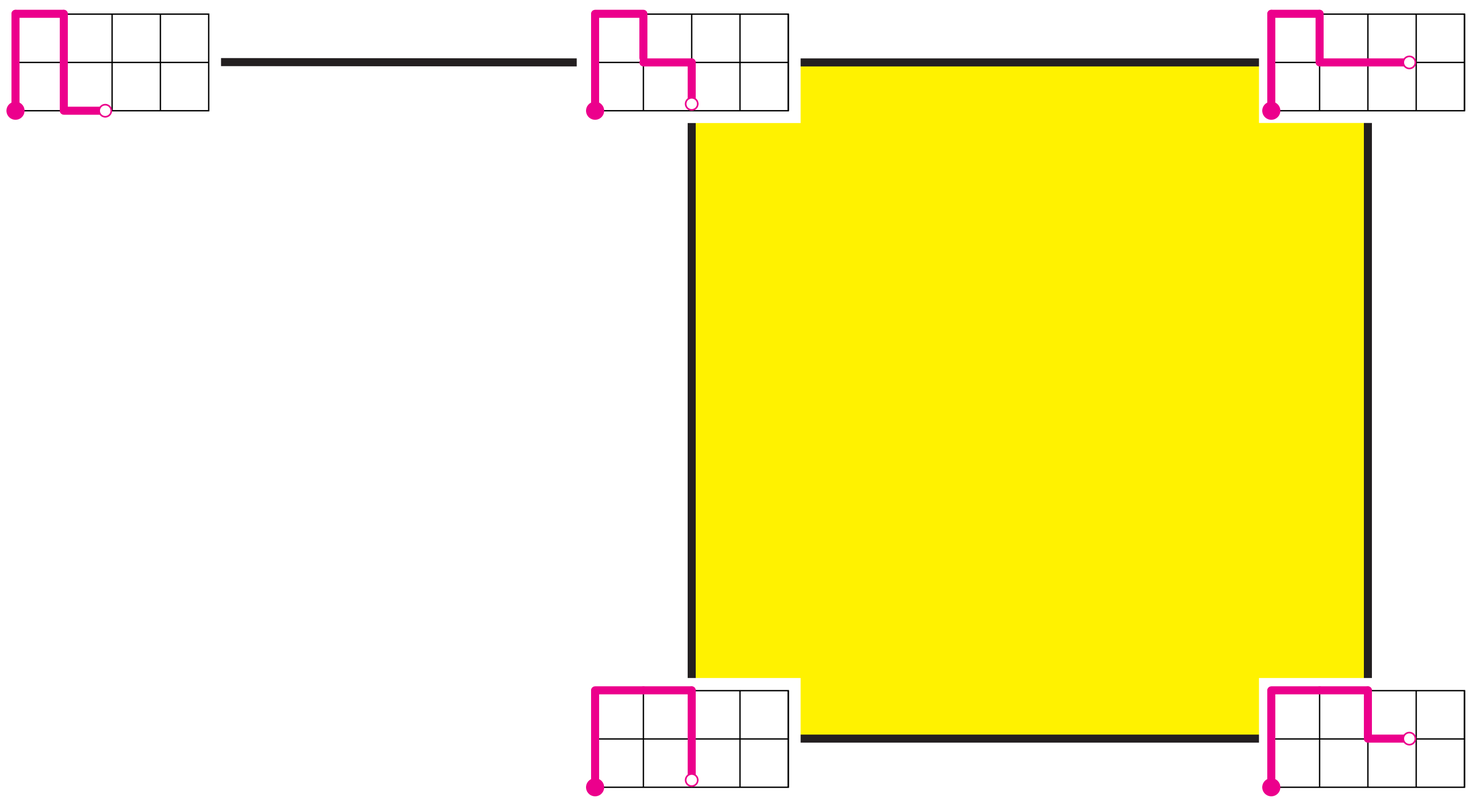


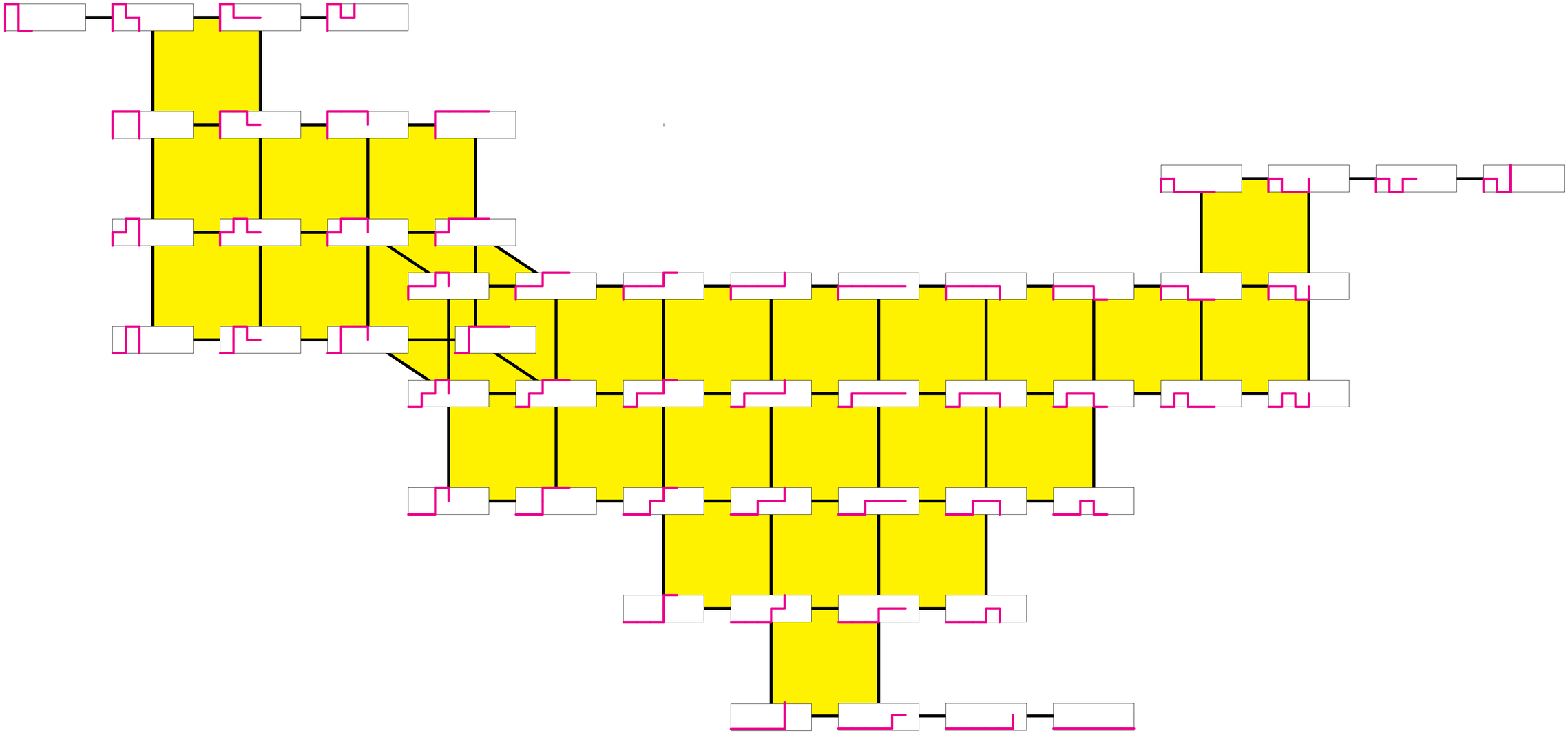
2. MAPS OF IMAGINARY WORLDS

Organizing complex information

Idea: Build the **map of possibilities.**



Idea: Build the **map of possibilities.**



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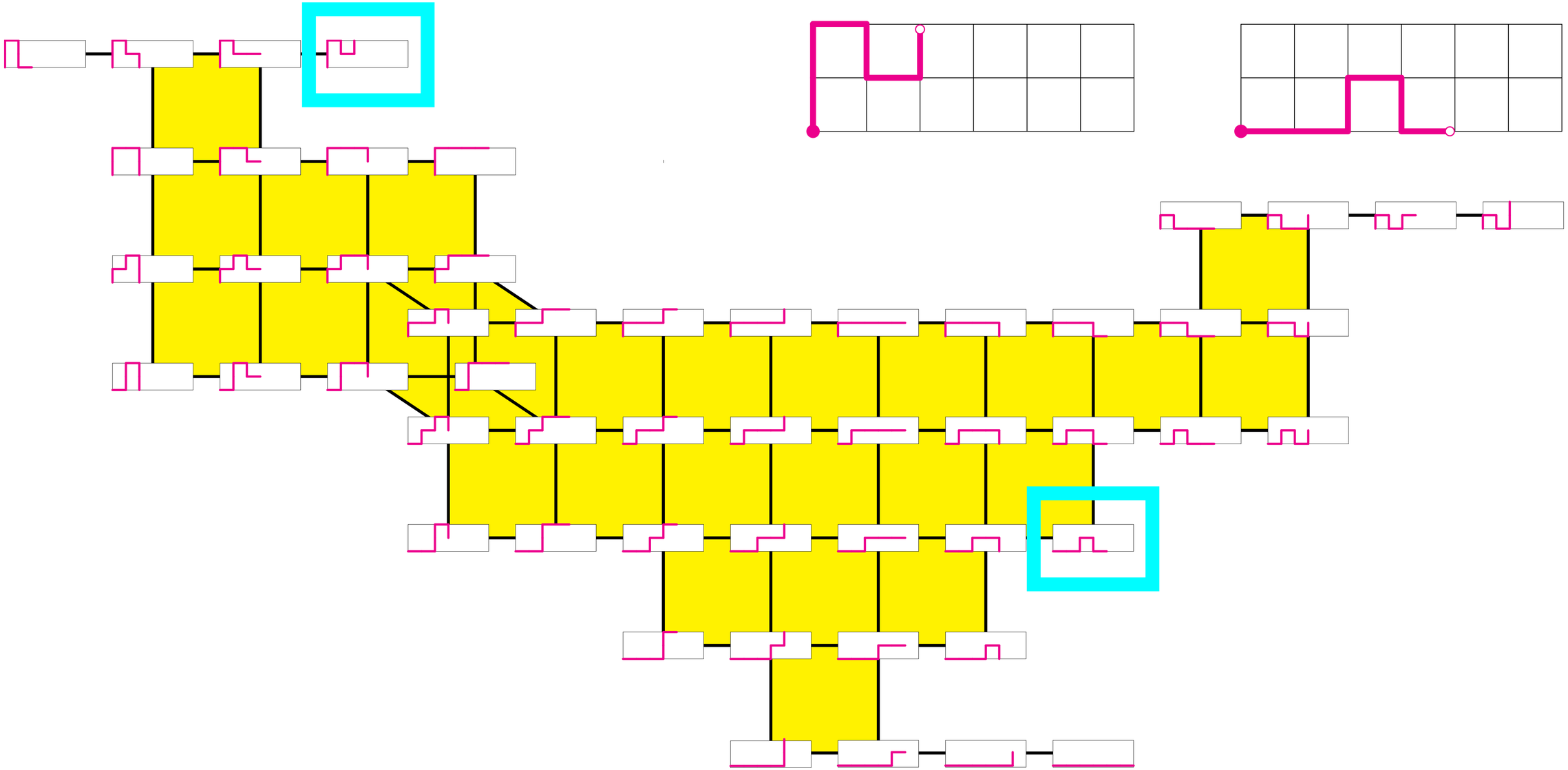

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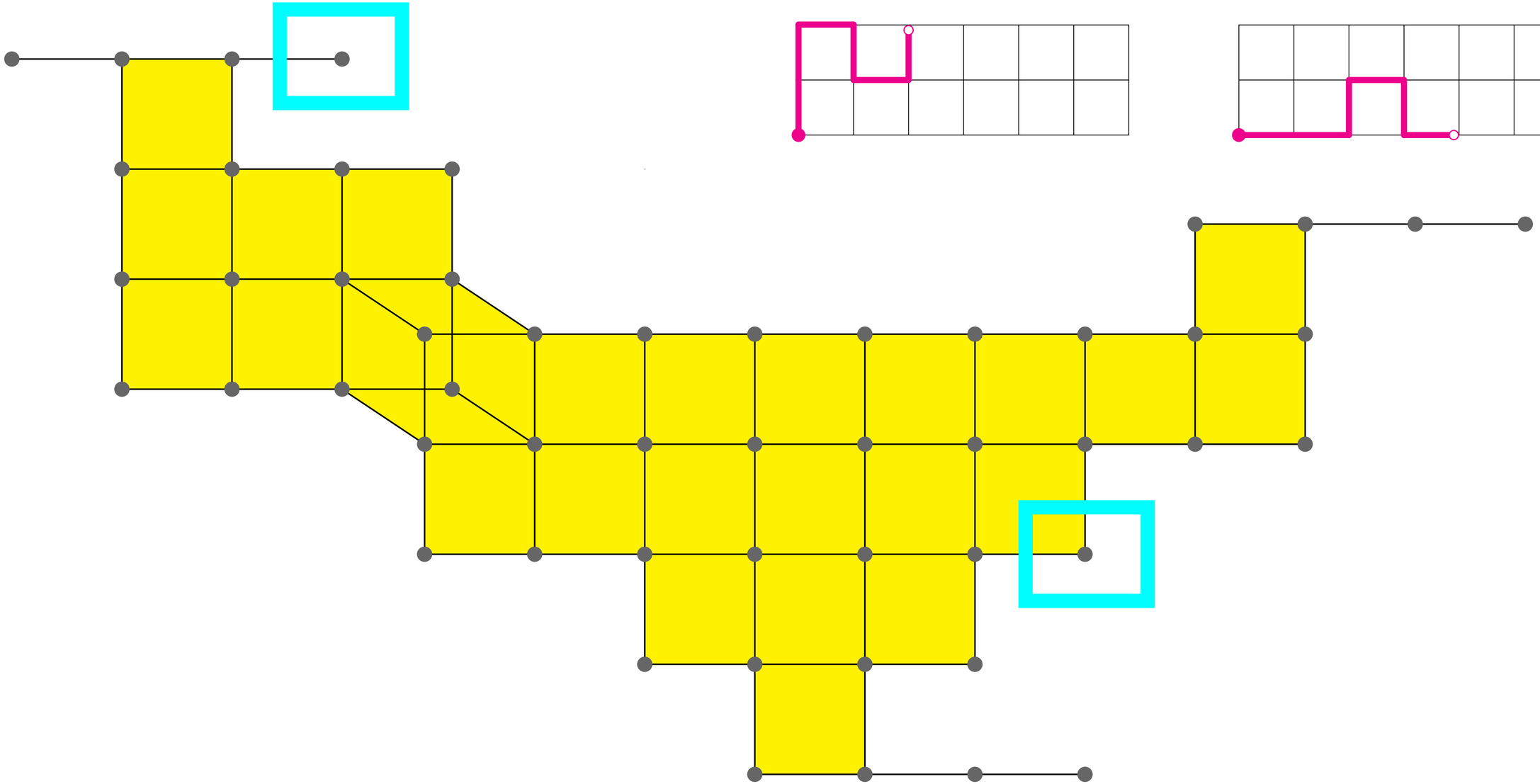

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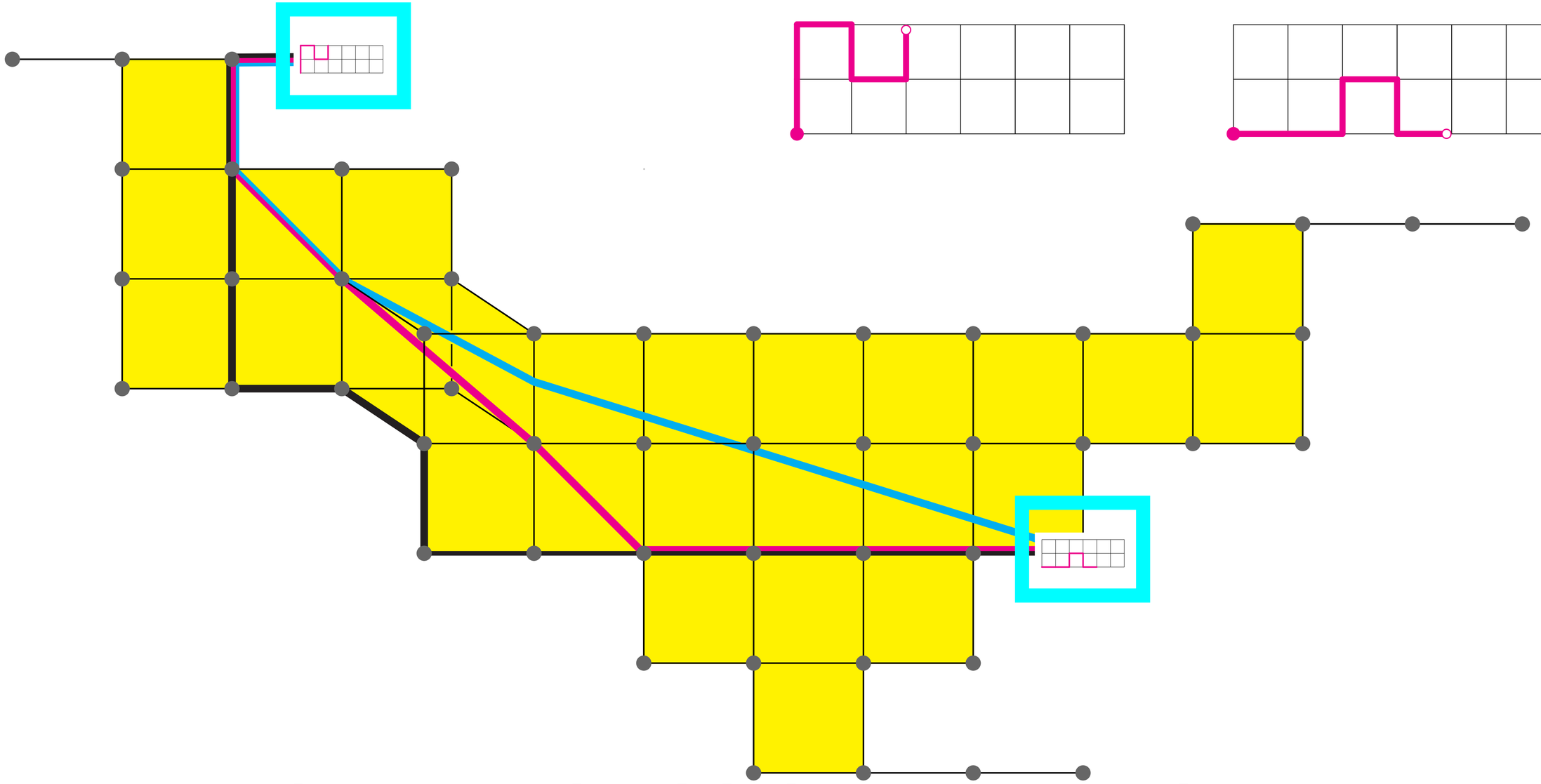
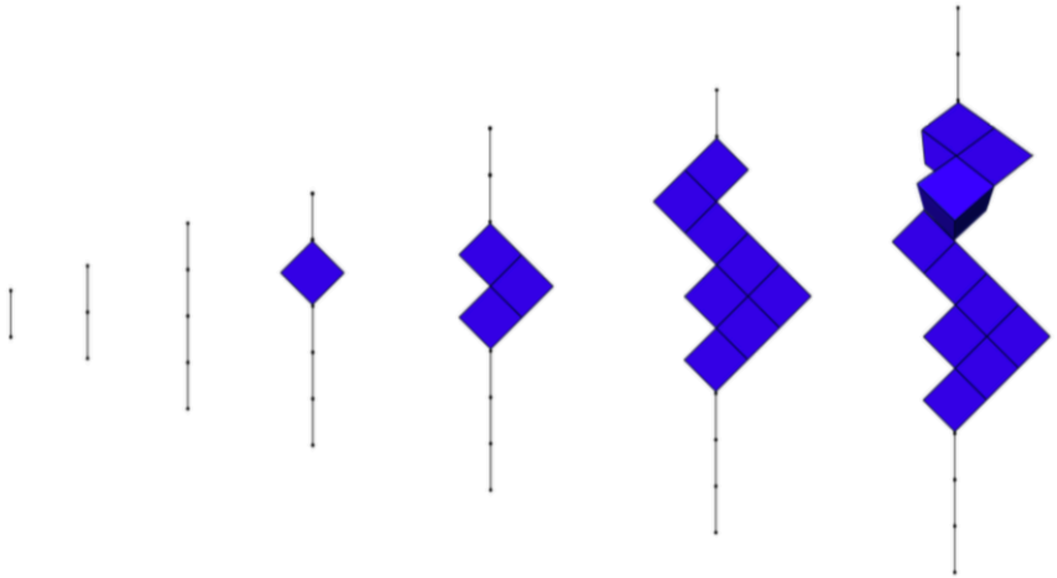


We “just” have to build the map and navigate it.

Difficulty: ¡These maps are gigantic!

Example. (Baker, Yatchak) The map of the robot in a 1 x n tunnel:



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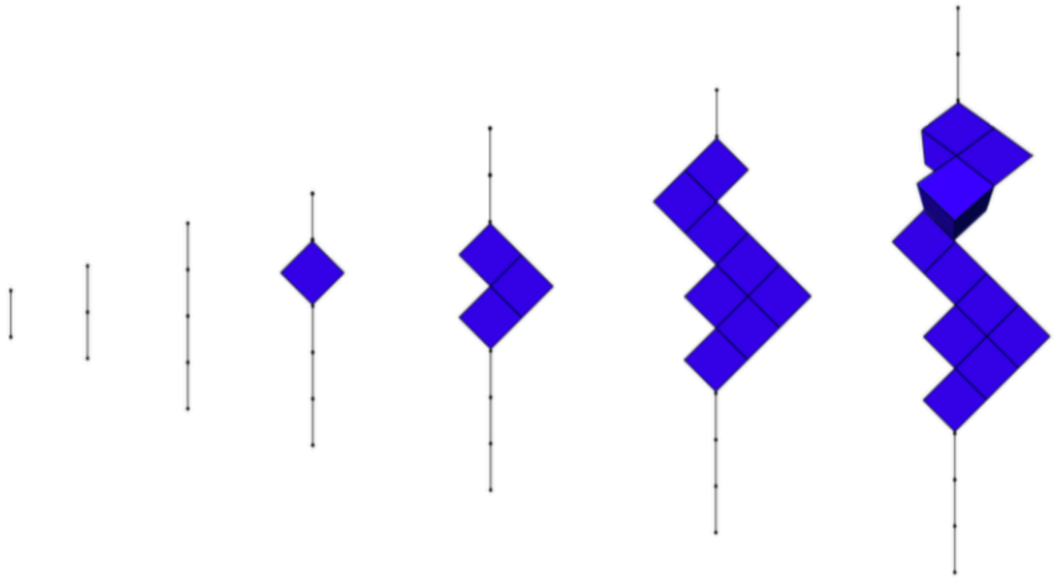
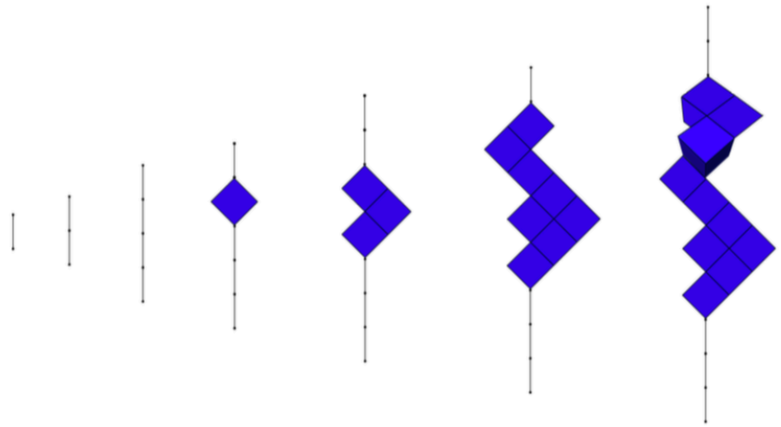


# of vertices: 2 3 5 8 13 21 34

Do you see a pattern? (Write it in the Chat box.)

Difficulty: ¡These maps are gigantic!

of vertices: 2 3 5 8 13 21 34



Good news? No…
The Fibonacci numbers are **gigantic.**

The robot of length 100 has
354’224,848’179,261’915,075 vertices and **34** dimensions

We won’t be able to do this by brute force. We need a good idea.

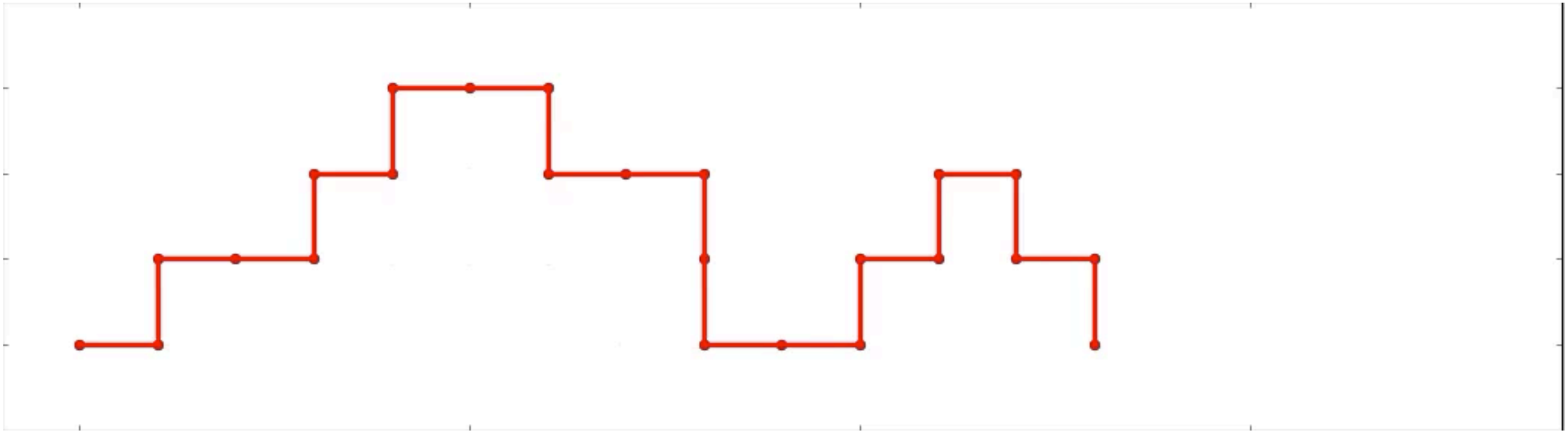
Theorem. (FA, Owen, Sullivant, 2011) If the map of a robot has negative curvature, we can move that robot optimally.

Theorem. (FA, Baker, Bastidas, Ceballos, Guo, Yatchak, 2014-17) The map of a robotic arm in a tunnel **does** have negative curvature.

Summary. We **can** move the robotic arm in a tunnel optimally!

We **can** move the robotic arm in a tunnel!

```
Enter the number of rows in the grid (grid height): 5
Enter a valid state:  ruruurddruururd
Enter a valid state:  rruurdrrrdruruu
The minimum number of steps is 26 .
The minimum number of individual moves is 67.
```



July 4, 2016

Three days later:

The Washington Post

The Switch

In an apparent first, Dallas police used a robot to deliver bomb that killed shooting suspect

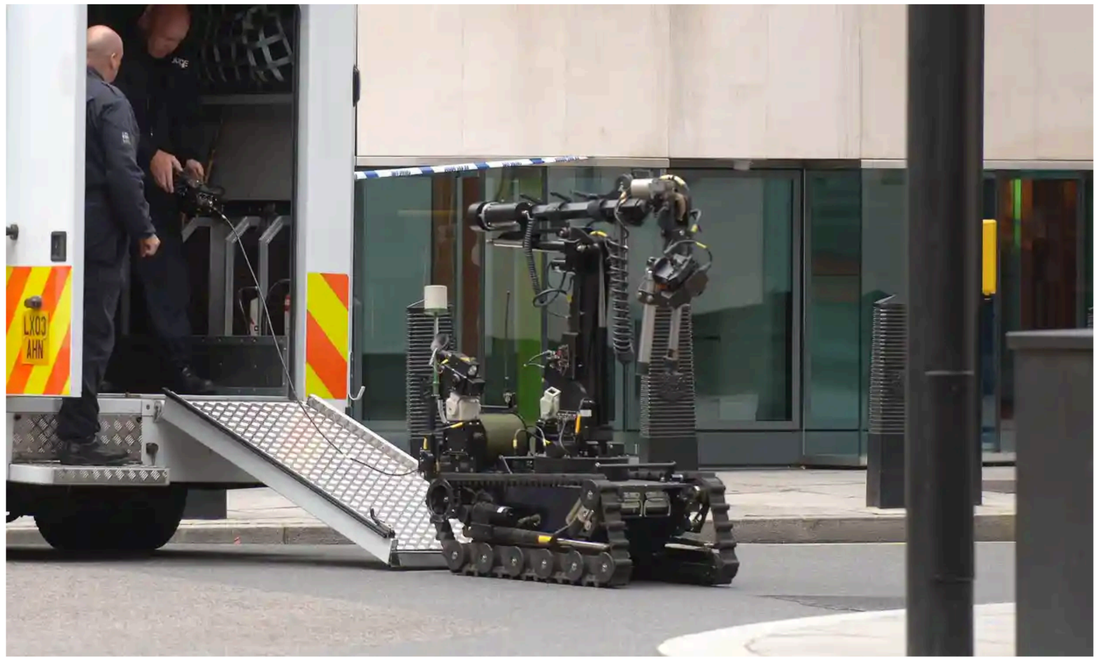
July 7, 2016

3. SOCIETY

Why move a robot?

Use of police robot to kill Dallas shooting suspect believed to be first in US history

Police's lethal use of bomb-disposal robot in Thursday's ambush worries legal experts who say it creates gray area in use of deadly force by law enforcement



▲ Bomb-disposal robots such as the one seen here have been used by the military as a weapon, according to Peter Singer of the New America Foundation, but never before by police. Photograph: Martin Godwin/The Guardian

What are they not telling us?

Use of police robot to kill Dallas shooting suspect believed to be first in US history

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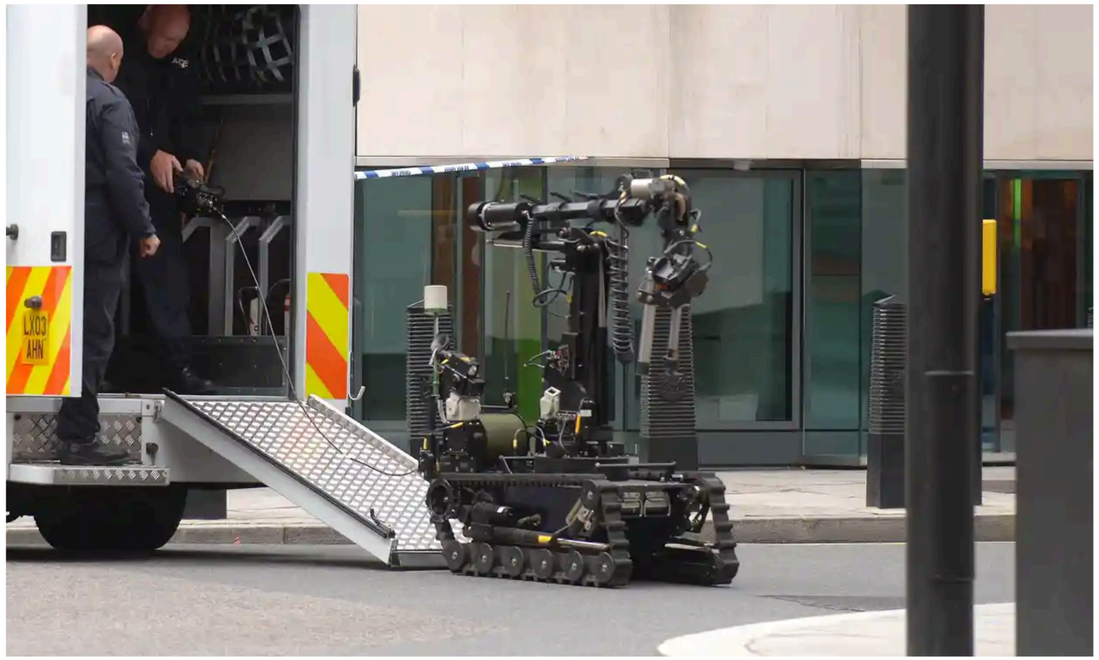
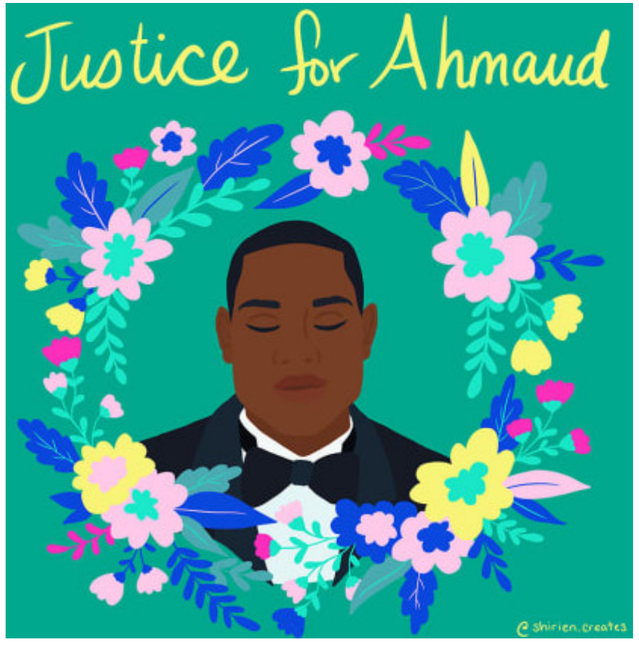
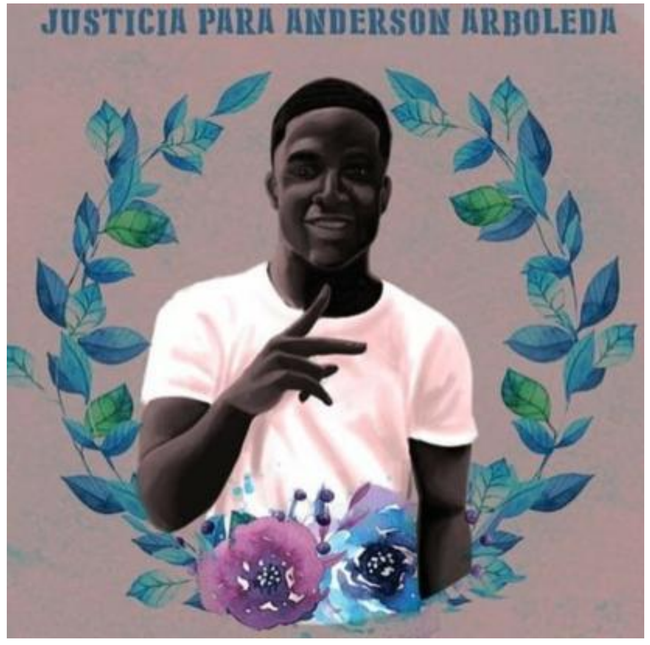


▲ Bomb-disposal robots such as the one seen here have been used by the military as a weapon, according to Peter Singer of the New America Foundation, but never before by police. Photograph: Martin Godwin/The Guardian

What are they not telling us?

This shooting took place at a Black Lives Matter protest.

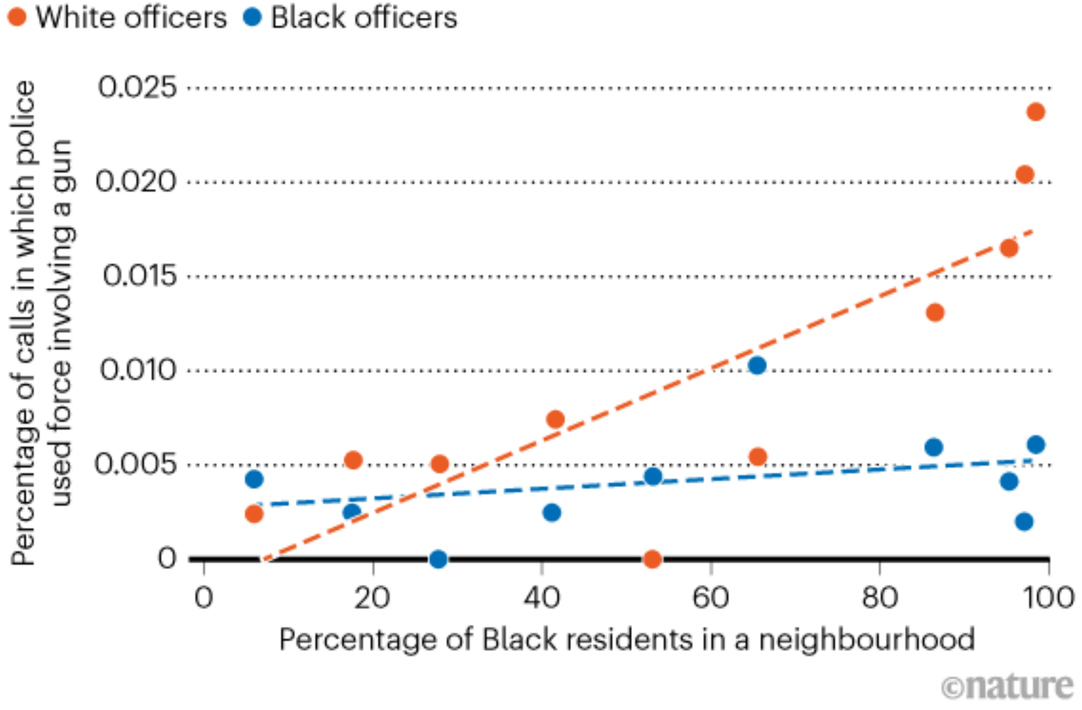




@shirien.creates

ANSWERING THE CALL

Researchers looked at responses to 1.2 million 911 emergency calls in a US city and plotted the use of force involving a gun across neighbourhoods, according to their racial composition. White officers were more likely to use a gun than were Black officers and more likely to do so in predominantly Black neighbourhoods.



What is our role in this?
TODXS CUENTAN
(Every one of us counts.)

Axiom 1. Mathematical potential is distributed equally among different groups, irrespective of geographic, demographic, and economic boundaries.

Axiom 2. Everyone can have joyful, meaningful, and empowering mathematical experiences.

Axiom 3. Mathematics is a powerful, malleable tool that can be shaped and used differently by various communities to serve their needs.

Axiom 4. Every student deserves to be treated with dignity and respect.

FA: Todxs Cuentan. Notices of the AMS, Nov 2016

What is our role in this?
TODXS CUENTAN

Axiom 3. Mathematics is a powerful, malleable tool that can be shaped and used differently by various communities to serve their needs.

– Who holds that power?

– How do we use it?

– Who do we share it with?

– Who pays for it? Why?

– Which communities benefit? Which are harmed?

What is our role in this?
TODXS CUENTAN

Axiom 3. Mathematics is a powerful, malleable tool that can be shaped and used differently by various communities to serve their needs.

Some great examples:

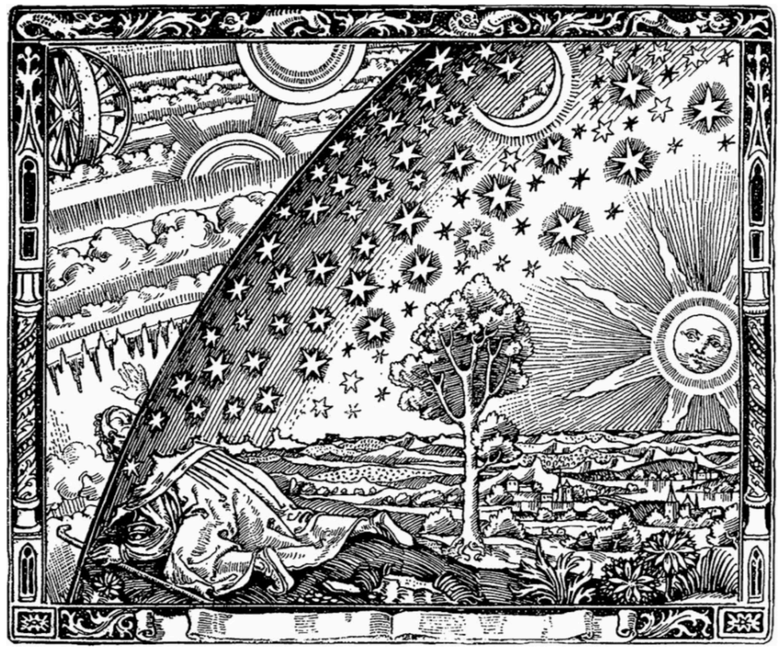
Black in AI (Rediet Abebe, Timnit Gebru)
(Twitter: @black_in_ai)

Data for Black Lives
(Twitter: @Data4BlackLives)

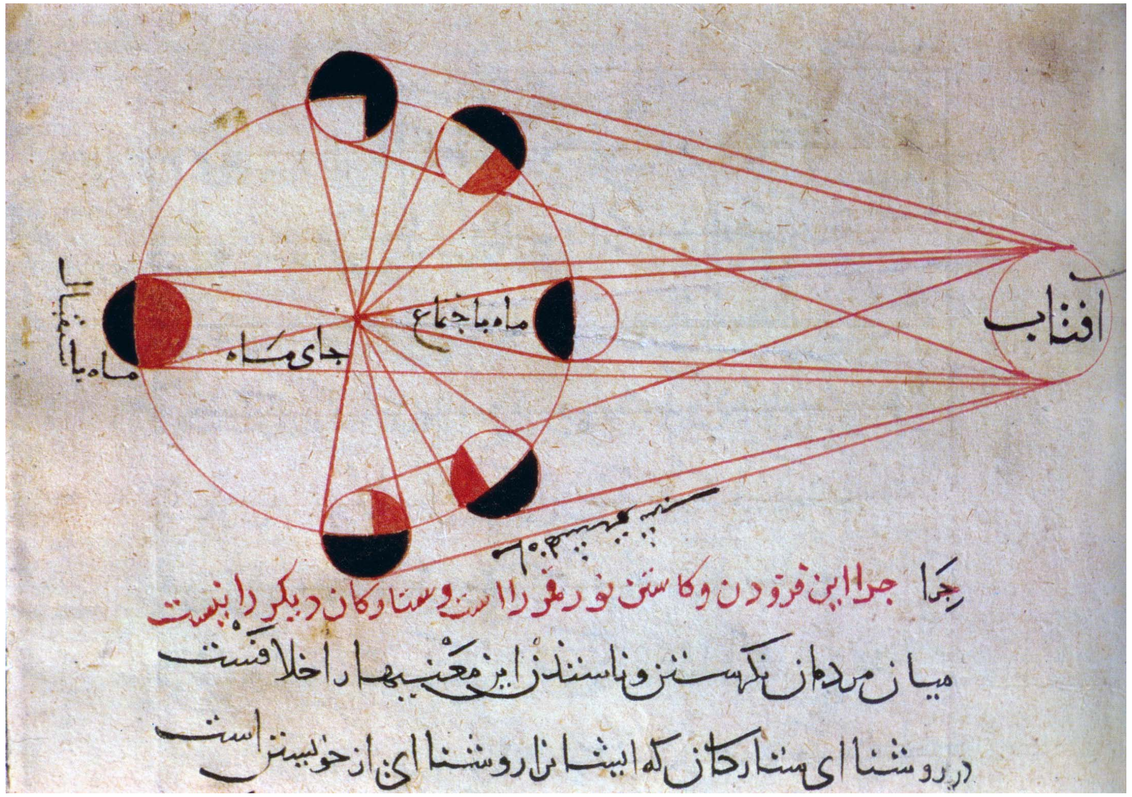
4. GEOMETRY

The curvature of the world

What is the shape of the Earth?



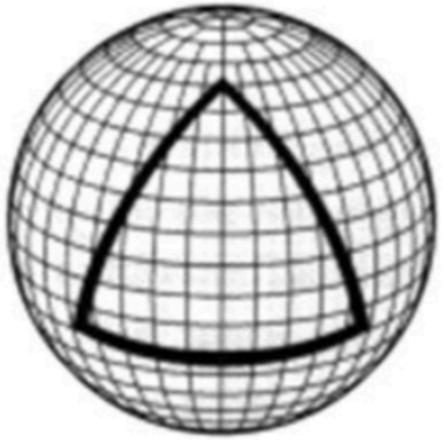
The Earth is round!

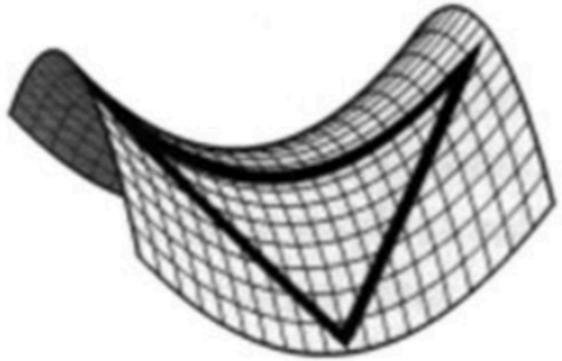


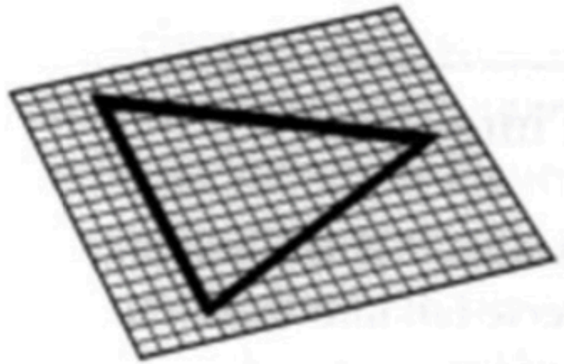
Eratosthenes

(Libya→Egypt: 276–194 A.C.)

CURVATURE







curvature:

positive negative zero

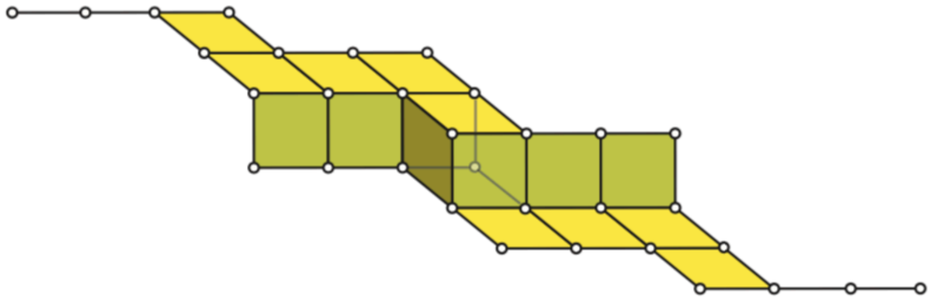
triangles:

fat thin flat

Theorem. If a space has negative curvature, any two points are connected by a unique shortest path.

Theorem. (FA, Owen, Sullivant, 2011) If a **cube complex** has negative curvature, we can find that shortest path.

Cube complex: A space made of cubes glued face-to-face.

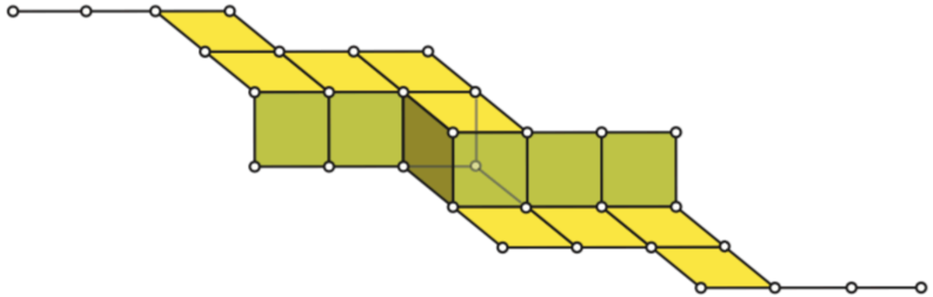


How do we know if the curvature is negative? (1)

In general, this question is hard! Do we measure every triangle?

Theorem. (Gromov) It is much easier to know if a **cube complex** has negative curvature. No need to measure!

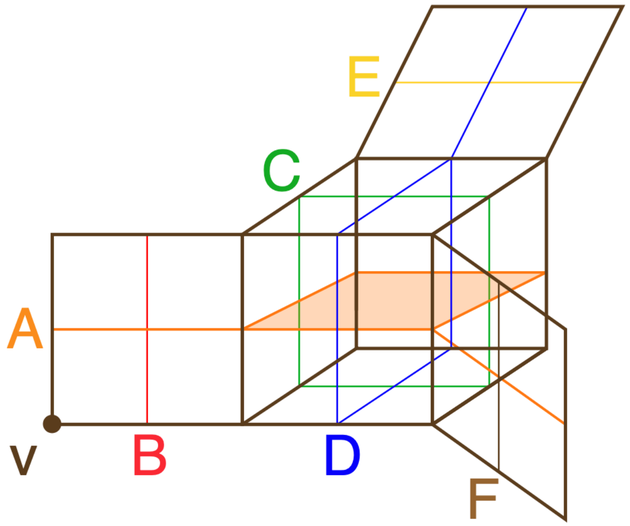
1. topology: no holes ("simply connected")
2. combinatorics: no empty cubes ("links are flag")



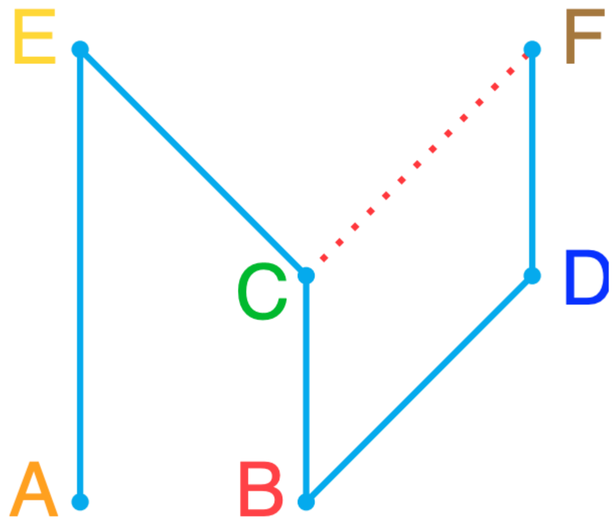
How do we know if the curvature is negative? (2)

Theorem. (FA, Owen, Sullivant, 2008) Cube complexes of negative curvature are those which have a "remote control".

"world"



"remote control"



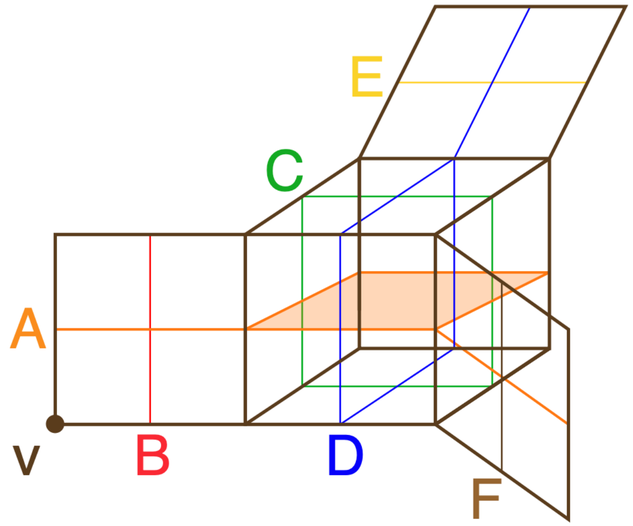
How do we know if the curvature is negative? (2)

Theorem. (FA, Owen, Sullivant, 2008) There is a bijection

rooted cube complexes of negative curvature

PIPs: posets with inconsistent pairs

"world"



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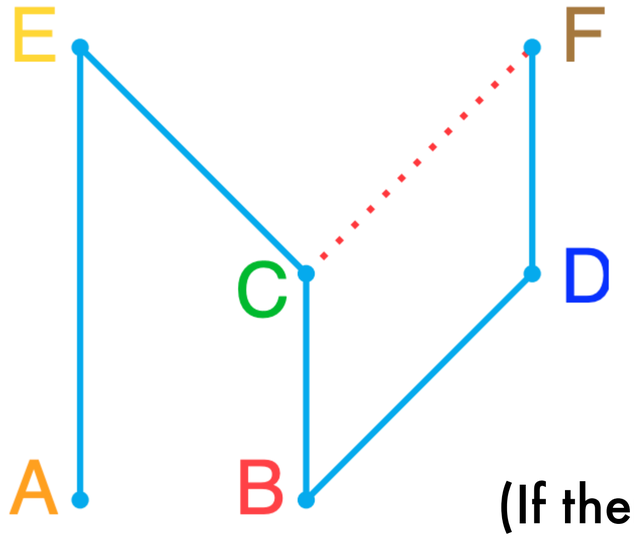
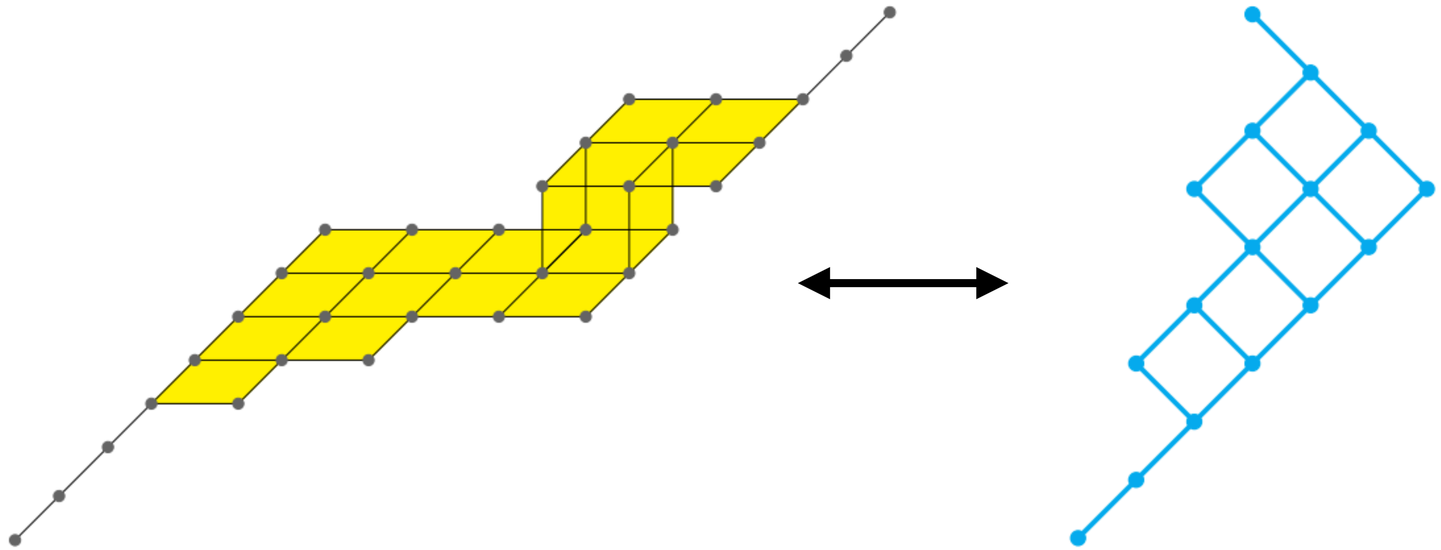


(If there is time, I'll tell you how this works at the end.)

Good news: To prove a cube complex has negative curvature, we "just" need to identify its remote control!

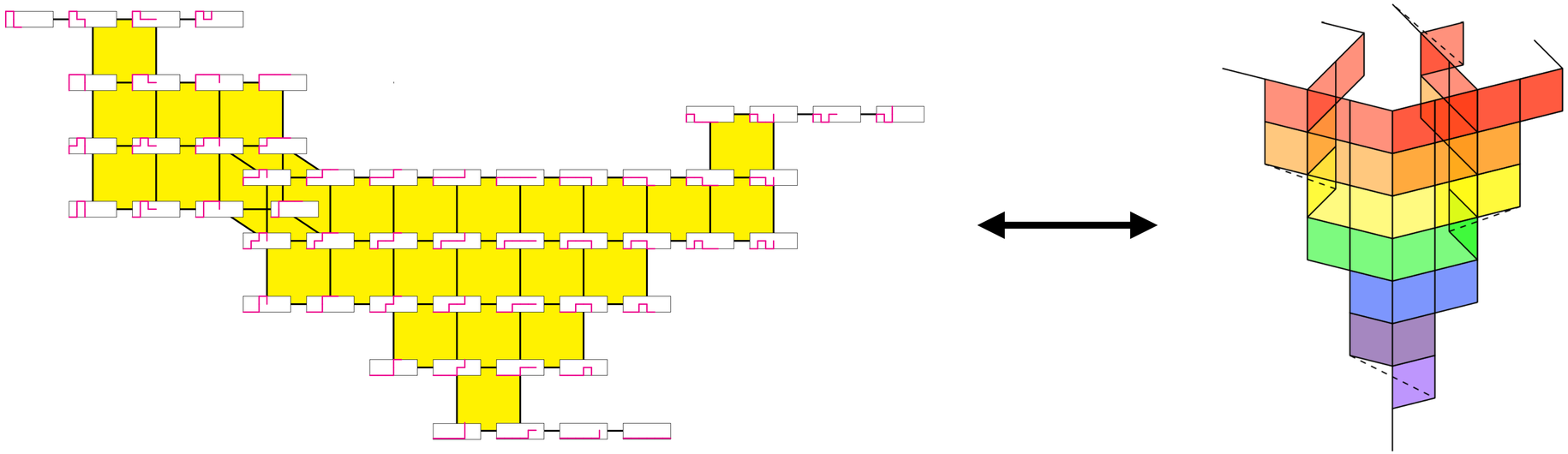
Theorem. (FA, Baker, Yatchak) For the robot in a 1 x n tunnel:



354'224,848'179,261'915,075 vertices
34 dimensions
(exponential growth)

251,001 vertices
2 dimensions
(quadratic growth)

Theorem. (FA, Baker, Bastidas, Ceballos, Guo, Yatchak)
The map of a robot in a rectangular tunnel has negative curvature.



Corollary. (FA, Baker, Bastidas, Ceballos, Guo, Yatchak)
We can move the robotic arm in a tunnel optimally!

We can move the robotic arm in a tunnel!

```
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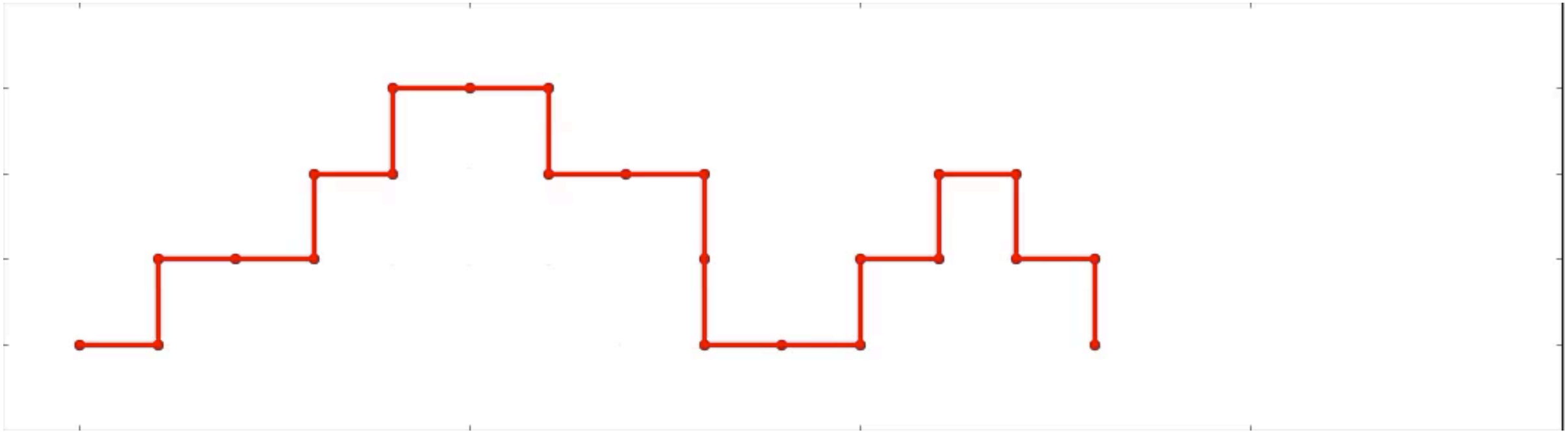
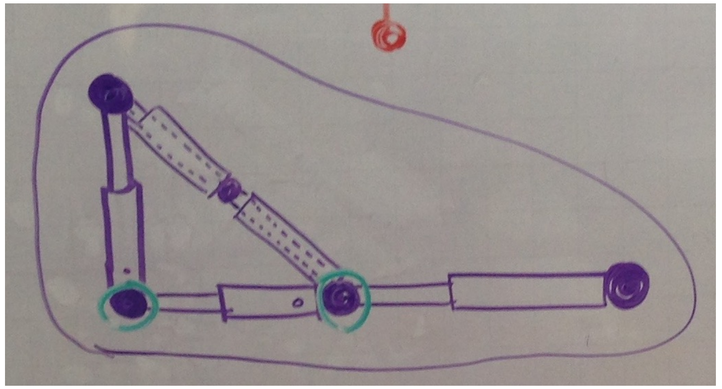


5. SOCIETY

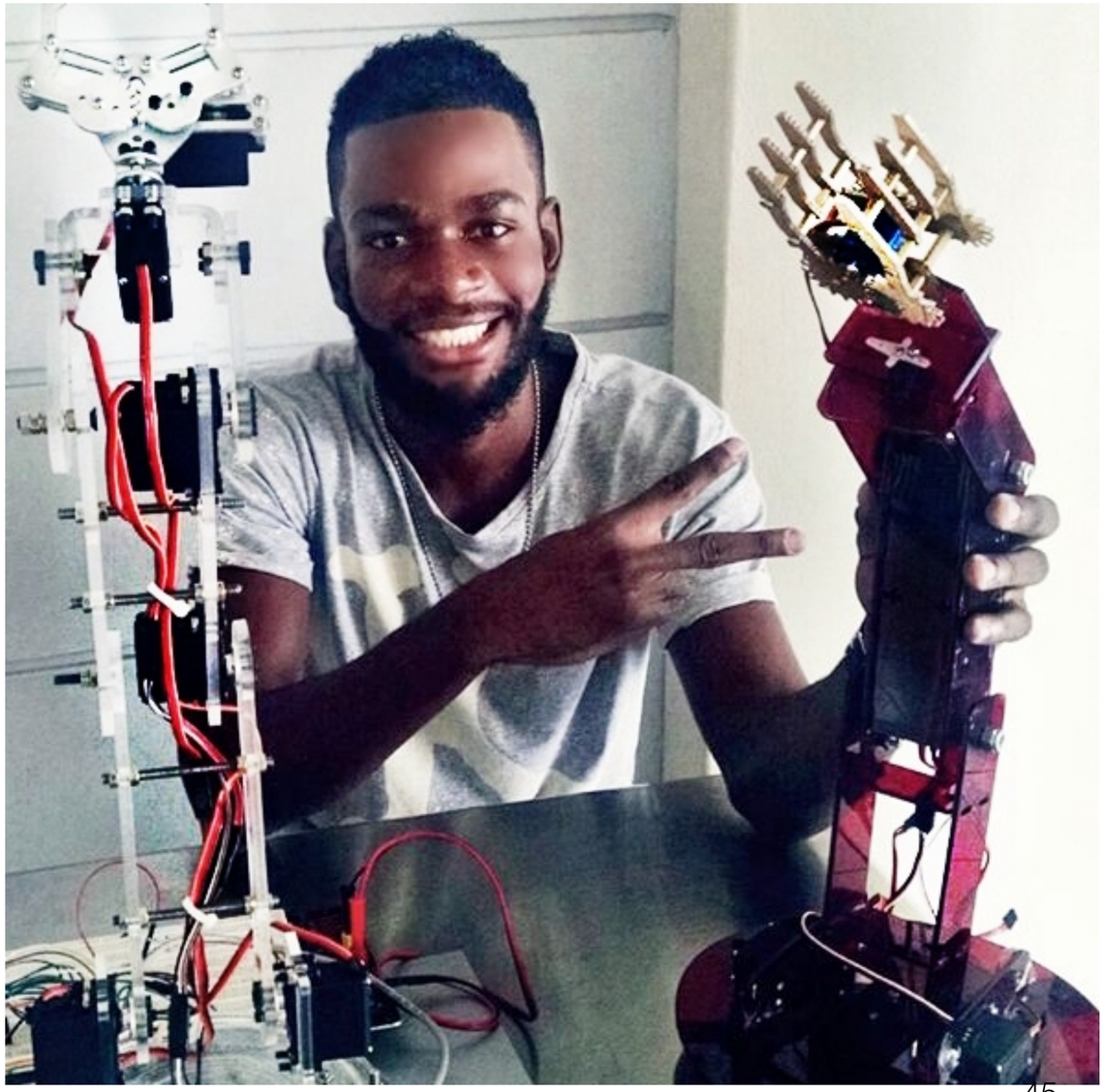
Why move a robot?

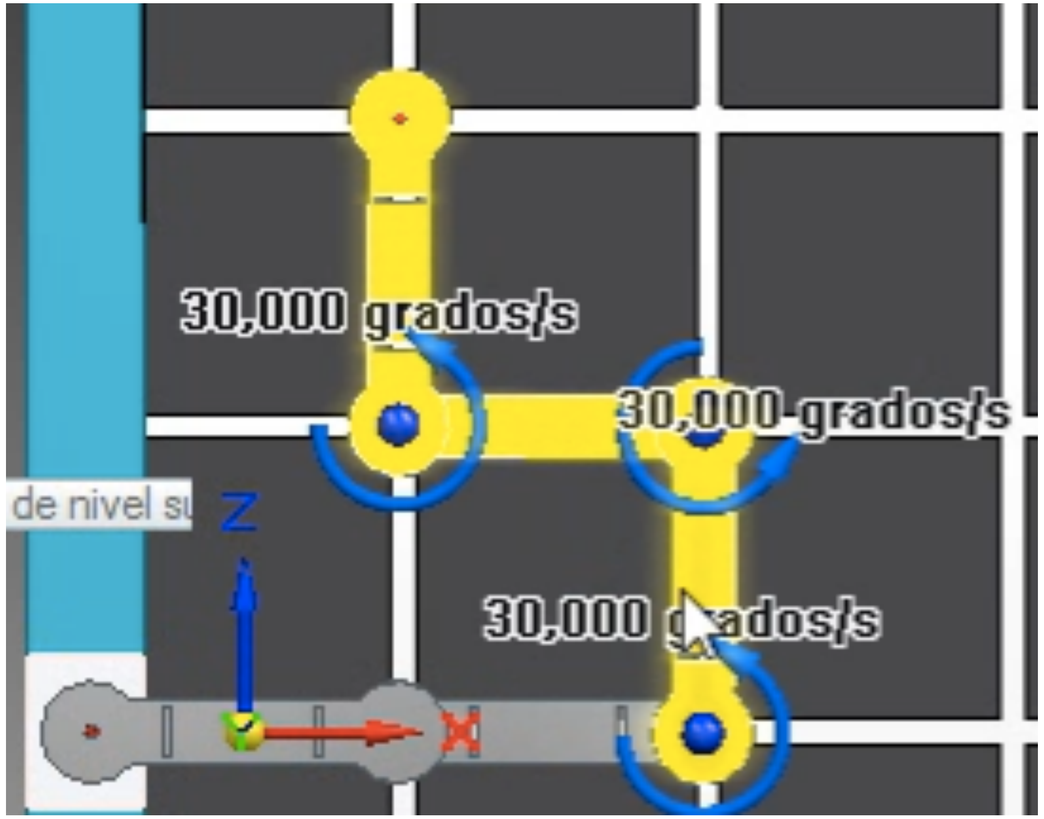
CLUBES DE CIENCIA COLOMBIA



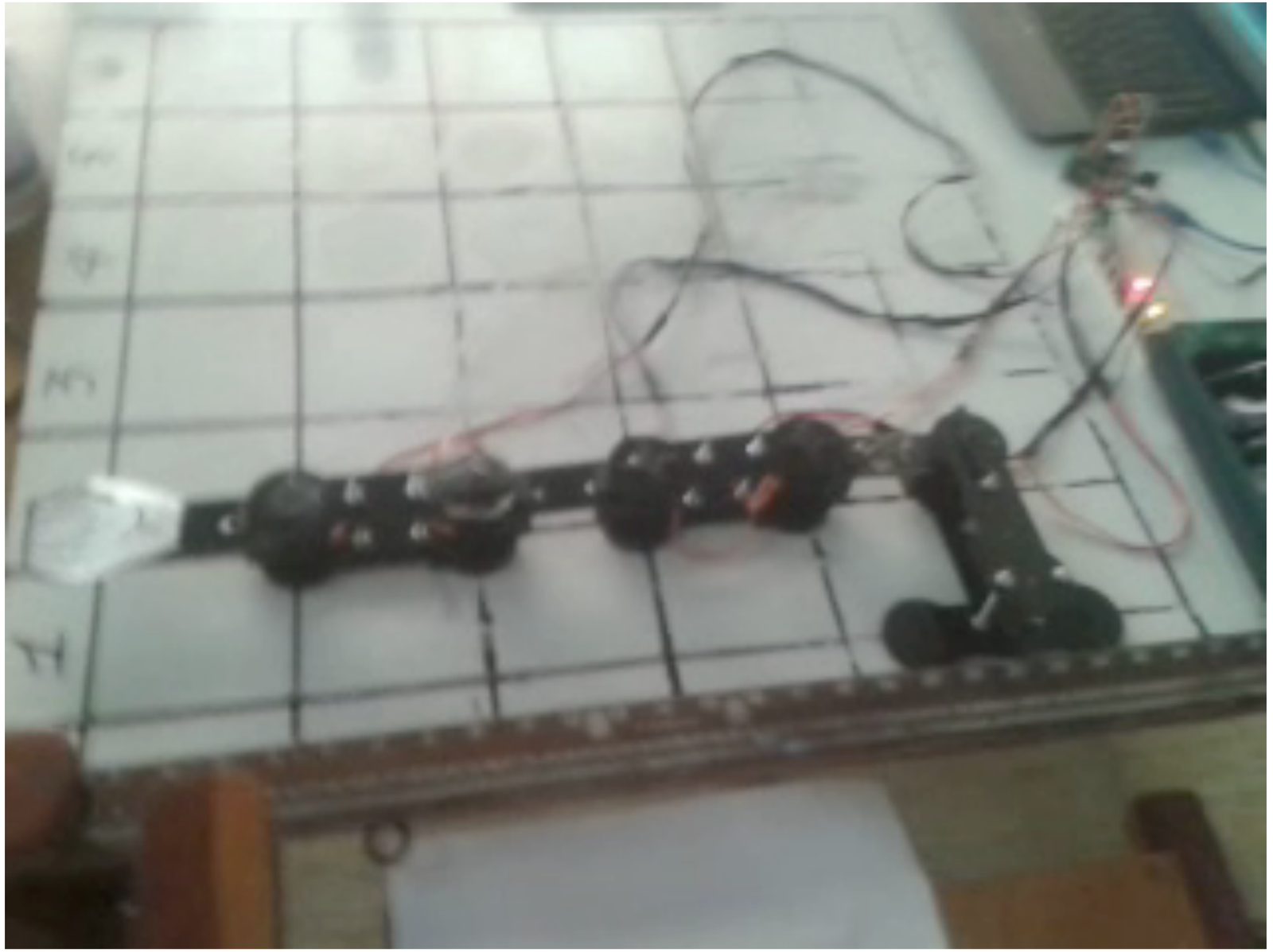
ITM, Medellín, 2016

CLUBES DE CIENCIA COLOMBIA





Arlys Asprilla, 2016



Arlys Asprilla, 2017

ESCUELA DE ROBÓTICA DEL CHOCÓ

Feb. 2020

ESCUELA DE ROBÓTICA DEL CHOCÓ

Innovation Girls

ESCUELA DE ROBÓTICA DEL CHOCÓ

ESCUELA DE ROBÓTICA DEL CHOCÓ

ESCUELA DE ROBÓTICA DEL CHOCÓ

ESCUELA DE ROBÓTICA DEL CHOCÓ

Deison Rivas
Juan David Cuenta

When I say “robot”, what robot do you imagine?

ROBOTS

When I say “robot”, what robot do you imagine?

Please write it in the Chat box.

¡¡¡ muchas gracias !!!

questions? comments? reactions?

(about the math, my personal trajectory, etc.)

CAT(0) Geometry, Robots, and Society

The paper is available at http://math.sfsu.edu/federico

Notices of the American Mathematical Society, August, 2020

6. TODXS CUENTAN

Mathematics and Community

TODXS CUENTAN

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FA: Todos Cuentan. Notices of the AMS, Nov 2016