Lecture Lecture 2 *54 min*. We prove Hilbert's basis theorem. Then we define monomial orderings, initial ideals, and Groebner bases. (My apologies for the technical difficulties, unfortunately the video has no sound!)

Lecture Lecture 4 *50 min*. How does one recognize a Grobner basis? How does one construct a Grobner basis? We discuss, Buchberger's criterion and algorithmm, which give nice and simple answers to these questions. Then we discuss minimal and reduced Grobner bases.

Lecture Lecture 5 *47 min*. We prove that every ideal I has a unique reduced Grobner basis. We discuss two applications: determining whether two ideals are equal, and solving systems of polynomial equations by elimination. (The camera starts moving after about 22 minutes, sorry!!!)

Lecture Lecture 6 *53 min*. We prove that you can use Grobner bases to compute elimination ideals, and to compute the intersection of two ideals in a polynomial ring.

Lecture Lecture 7 *53 min*. Computing syzygies (linear relations with polynomial coefficients) between polynomials is easy using Grobner bases. I state Schreyer's theorem, and review some basic facts about modules.

Lecture Lecture 8 *53 min*. I describe term orders and Grobner bases in a free module F over a polynomial ring. I state and begin to prove that if G is a Grobner basis for a submodule of F then the "basic syzygies" of G generate the module of all syzygies of G; and in fact, they are a Grobner basis for it.

Lecture Lecture 9 *53 min*. We complete the proof of Schreyer's theorem, and begin to discuss free resolutions of modules.

Lecture Lecture 11 *53 min*. We define free resolutions of modules and prove Hilbert's syzygy theorem.

Lecture Lecture 12 *52 min*. We define the Hilbert functions and series of graded rings and modules, and compute some examples.

Lecture Lecture 13 *53 min*. We discuss finely graded modules and their Hilbert series, and carefully carry out some examples.

Lecture Lecture 14 *51 min*. We show how to compute the Hilbert series of a module from a resolution, and discuss several applications of these ideas.

Lecture Lecture 15 *54 min*. We prove Macaulay's theorem that M and in(M) have the same Hilbert series, and state his characterizations of Hilbert series of graded rings satisfying some mild hypotheses. (The microphone ran out of battery at 43:30, sorry! You can still hear what I'm saying if you turn the volume up.)

Lecture Lecture 16 *53 min*. We begin the study of monomial ideals. We prove the correspondence between squarefree monomial ideals and simplicial complexes, and illustrate with examples.

Lecture Lecture 17 *51 min*. We give a decomposition of Stanley-Riesner ideals into prime ideals, and prove a formula for the Hilbert series of Stanley-Reisner rings.

Lecture Lecture 18 *51 min*. We prove the formula for the coarse Hilbert series of a Stanley-Reisner ring. Then we discuss the general idea behind the field of algebraic topology.

Lecture Lecture 19 *53 min*. We define the homology groups of a simplicial complex, and compute an example.

Lecture Lecture 20 *52 min*. We continue to discuss some general facts about homology and compute more examples.

Lecture Lecture 21 *51 min*. We state two descriptions of the graded Betti numbers of a monomial ideal M; one in terms of Tor(M,F), and another one in terms of simplicial complex homology.

Lecture Lecture 22 *52 min*. We prove the Tor characterization of Betti numbers, and begin to prove the homological interpretation of them.

Lecture Lecture 23 *51 min*. Computing the Betti numbers of a monomial ideal I is equivalent to computing the homology of the upper Koszul complexes of I. For squarefree I, Hochster's theorem tells us that this is just the homology of the links of the Alexander dual simplicial complex.

Lecture Lecture 24 *53 min*. We discuss algebraic and topological properties of Alexander duality, and use them to state Hochster's theorem, which describes the Betti numbers of the Stanley-Reisner ring of a simplicial complex in terms of the cohomology of its links.

Lecture Lecture 25 *51 min*. We see how the general linear group GL (and its Borel subgroup B and torus subgroup T) acts on the ring of polynomials, and discuss the ideals of R which are invariant under these actions. The GL-fixed ideals are the powers of the maximal ideal. The T-fixed ideals are the monomial ideals. The B-fixed (or "Borel fixed monomial ideals") are a very nice family of non-squarefree monomial ideals that we will discuss next time.

Lecture Lecture 26 *53 min*. We discuss how Borel-fixed ideals occur as generic initial ideals. We compute their Hilbert series and discuss other nice properties.

Lecture Lecture 29 *22 min*. We discuss polytopes, polyhedral complexes, and their chain complexes.

Lecture Lecture 30 *49 min*. We show a way to find a free resolution of a monomial ideal I "by picutre", using a labelled polyhedral cell complex X. We characterize the labelled cell complexes that support such a cellular resolution, and describe the Hilbert series of I in terms of the graded Euler characteristic of X.

Lecture Lecture 31 *52 min*. We show how the Betti numbers of a monomial ideal can be obtained from a cellular resolution.

Lecture Lecture 32 *52 min*. We discuss two examples of cellular resolutions: Taylor resolutions, and permutahedron resolutions.

Lecture Lecture 33 *49 min*. We discuss the hull resolution, a cellular resolution of an arbitrary monomial ideal in n variables which has length at most n.

Lecture Leture 34 *49 min*. We begin by discussing how to view hull complexes for artinian monomial ideals. Then we discuss generic monomial ideals.

Lecture Lecture 35 *53 min*. Every monomial ideal I in n variables has a cellular resolution by a simplical complex, of length at most n. When I is generic, the Scarf complex of I provides a minimal free resolution. When I is not generic, the Scarf complex of a generic deformation of it gives a resolution.

Lecture Lecture 36 *51 min*. We begin to discuss semigroup algebras and lattice ideals.

Lecture Lecture 37 *50 min*. We prove that a semigroup ring is isomorphic to the quotient of the polynomial ring by the lattice ideal, and offer several characterization of affine semigroups.

Lecture Lecture 38 *42 min*. We show that a semigroup has a unique minimal set of generators. For saturated semigroups this is called the Hilbert basis. Then we discuss some interesting properties of semigroups of integers.

Lecture Lecture 39 *50 min*. We begin to study initial ideals and initial complexes of lattice

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ideals.

Lecture Lecture 40 *53 min.* We describe the initial ideal of a lattice ideal in terms of an optimization problem in integer programming. We also describe the initial complex of a lattice ideal in terms of regular subdivisions.

Lecture Lecture 41 *50 min.* We present a topological/combinatorial formula for the graded Betti numbers and the Hilbert series of a lattice ideal.