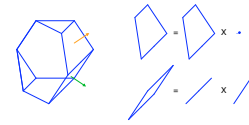




$$\begin{array}{ccc}
 A \otimes A & \xrightarrow{\varphi} & A & \xrightarrow{\Delta} & A \otimes A \\
 \downarrow \Delta \otimes \Delta & & & & \uparrow \varphi \otimes \varphi \\
 A \otimes A \otimes A & \xrightarrow{A \otimes T \otimes A} & A \otimes A \otimes A & & A \otimes A \otimes A
 \end{array}$$



hopf algebras and combinatorics

san francisco state university . universidad de los andes

federico ardila m.

The topic. We will study Hopf algebras, with an emphasis on their connections with combinatorics. Hopf algebras are spaces with a notion of *multiplication* and *comultiplication*, which satisfy certain compatibility relations. They can loosely be thought of as generalizations of groups. They were defined by topologists in the 40s, and became an object of intensive study in their own right. Beginning in the 70s, combinatorialists found that many of their favorite objects can be naturally turned into Hopf algebras. This key observation gives us a rich source of interesting examples of Hopf algebras, as well as an elegant machinery to prove purely combinatorial theorems.

Instructor. Federico Ardila . federico@sfsu.edu . Thornton 927 . 1.415.338.7720

Logistics. This course is part of the SFSU-Colombia Combinatorics Initiative, a teaching and research collaboration between San Francisco State University and the Universidad de Los Andes in Bogotá, with guest participants from UC Berkeley and other sites. This course will be offered jointly at both locations; most lectures will take place at SFSU, and videos will be broadcast at Los Andes and freely available online. Students will have the opportunity to discuss the course material and assignments in an internet forum.

Homework. During the first half of the course there will be biweekly assignments. These will range from fairly routine exercises to challenging problems, and working in groups will be encouraged.

Project. In the second half of the course students will write a final project. This will be done in pairs, and international groups will be encouraged. The project is a chance to go much deeper into a topic of your choice. This could be an expository paper summarizing an aspect of Hopf algebras that interests you, the beginning of an original research project, or (why not?) the solution to an open problem in the field. This may be a good opportunity to find a thesis topic. I will suggest possible projects.

Language. The class will be taught in English. (Ustedes entenderán...) You may write your assignments in English (even if your English isn't great but you want to practice) or in Spanish.

Textbook.

There is no required textbook. I will provide lecture notes. Parts of the course will follow parts of:

- M. Sweedler. *Hopf algebras*
- S. Montgomery. *Hopf algebras and their actions on rings*.
- M. Aguiar. *Monoidal functors, species, and Hopf algebras*.

as well as other sources, which I will announce as they become relevant.

Course website.

<http://math.sfsu.edu/federico/hopf.html>

You are expected to visit this website often, and participate actively on the online discussion forum, which should be a very useful resource. On the website you will find, among others, links to the homework assignments, some suggested final projects, the lecture notes, the lecture videos, and the online discussion forum.

Meetings.

.sf. Tue, Thu, 9:35-10:50. Thornton 211. .bog. Wed, Fri 5:00-6:20.

Office hours.

To be determined.

Prerequisites.

You must be prepared to devote at least 10 hours a week on this class, and have a solid knowledge of groups and rings. The only formal prerequisite is:

- Math 435/735 (SFSU) or Algebra Abstracta 2 (Los Andes) or equivalent.

Grading and tentative due dates.

- 40% main homework (due Feb. 7, Feb 21, Mar 6, Mar 20)
- 10% light homework (in April, May)
- 10% project proposal (due Apr 3)
- 40% final project (due May 18)

Extra credit:

- 10% in the final grade for active participation in online forum.
- 10% in the project grade for teams of a bog and an sf student.