PROPOSITION 3.1.1. The following are equivalent:

1) M and u are coalgebra maps

2) Δ and ϵ are algebra maps

3) a) $\Delta(1) = 1 \otimes 1$

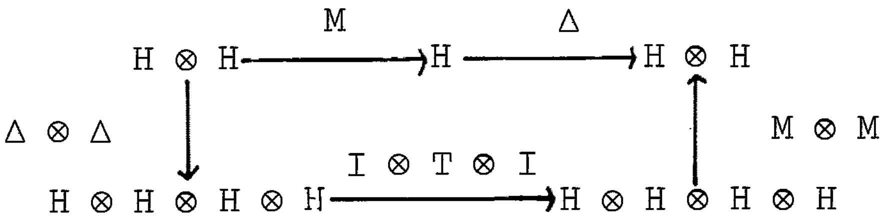
b) $\Delta(gh) = \sum_{(g),(h)} g_{(1)}h_{(1)} \otimes g_{(2)}h_{(2)}$

c) $\epsilon(1) = 1$ and

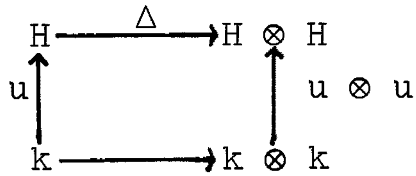
d) $\epsilon(gh) = \epsilon(g)\epsilon(h)$

<u>Proof</u>. 2) and 3) are obviously equivalent.

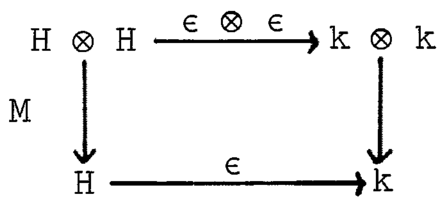
The equivalence of 1) and 2) may be seen by considering the following diagrams:



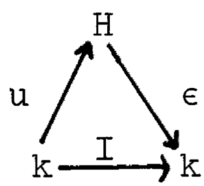
α)



β)



λ)



ρ)

The commutativity of α) and β) says exactly that Δ is an algebra map, whereas the commutativity of λ) and ρ) says ϵ is an algebra map. On the other hand α) and λ) commute if and only if M is a coalgebra map, and β) and ρ) commute in case u is a coalgebra map. Thus 1) is equivalent to 2).