1. **(Dull sequences) A sequence of positive integers is dull if for any $k > 1$ which appears in the sequence, the number $k > 1$ appears at least once before the first occurrence of$k$.**

   (a) **Find $a_n(m)$, the number of dull sequences of length n where the largest number is m.** We show our desired number by first seeing the correspondence dull sequences have with set partitions.

   The constraint we are given tells us that if the number k appears, we must have that $1, 2, ..., k-1$ must also appear to the left of $k's$ first appearance. For a given dull sequence of length n with a largest number m, we show a 1-1 correspondence to a $m-$set partition of $[n]$. Let $\sigma = \sigma_1 ... \sigma_n$ be a dull sequence of length n with largest number m. Then we know σ contains the numbers $1, ..., m$. To get its corresponding set partition $A_1 \sqcup ... \sqcup A_m$ we define the sets by

   $$A_i = \{j : \sigma_j = i\}$$

   For example consider the dull sequence $\sigma = 11123223114444255$. Since the length of σ is 17 it must correspond to a set partion of [17]. By definition of A_i we get that partition

   $$\{1, 2, 3, 9, 10\} \sqcup \{4, 6, 7, 15\} \sqcup \{5, 8\} \sqcup \{11, 12, 13, 14\} \sqcup \{16, 17\}$$

   Thus we see that for each set A_i its minimum value corresponds to the first appearance of i. Therefore given any set partition of $[n]$, order the sets A_i in increasing order by its minimum values. Do the reverse to what we did to sets A_i to create a sequence. By construction this will have to be a dull sequence.

   Hence we have that

   $$a_n(m) = S(n, m)$$

   where $S(n, k)$ is the Stirling number of the second kind.