We first check that permutations that are fixed may only have cycles of length at most 2. For example suppose one has a cycle of length three and after the transformation we get a permutation

$$\begin{array}{ccc} n & n+1 & n+2 \\ a & b & c \end{array}$$

then n maps to a, c maps to a so $n = c$. $n+1$ maps to b and a maps to b so $n+1 = a$. $n+2$ maps to c and b maps to c so $b = n+2$. Then (a, b, c) $= (n+1, n+2, n)$ which is not in standard form so it can't happen. for the case $n = 1$ there is only one permutation and it is fixed. for the case $n = 2$ there are two permutations, the identity $(1)(2)$ which is fixed, and the transposition (21) which is also fixed so the first number is 1 and the second is 2. Now we check the number follows the Fibonacci recurrence. If the permutation end with a cycle of length 1, this cycle has to be (n) because of the standard representation, and so n is mapped to n in cycle notation and after the transformation n is also mapped to n so before (n) we need a permutation of $[n-1]$ that is fixed under the transformation.

Now if the permutation ends in a cycle of length 2 then such cycle starts with n and because it is fixed, $n-1$ must be mapped to n and because the cycle has length 2, n must be mapped to $n-1$, then the permutation ends with the cycle $(n, n-1)$ and before this cycle we need a permutation of $[n-2]$ that is fixed under the transformation. This is the Fibonacci recurrence.