Pf

(I) These Coxeter groups are finite.

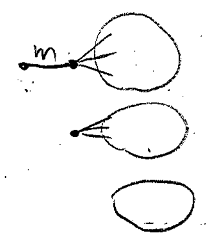
Prove (ppal minors of these A_Γ) > 0

Note: $\begin{pmatrix}\text{ppal minor} \\ \text{of } A_\Gamma\end{pmatrix} = \det A_{\text{subgraph of } \Gamma}$

Note: $\det A_{\Gamma_1 \sqcup \Gamma_2} = \det A_{\Gamma_1} \det A_{\Gamma_2}$

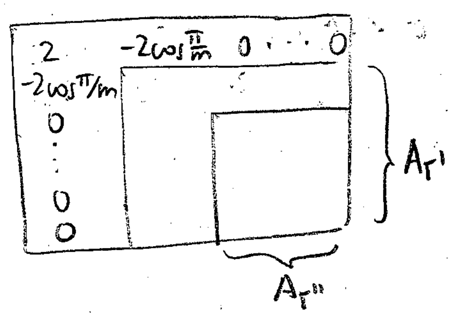
So enough to compute $d(\Gamma) = \det 2A_\Gamma > 0$.

Lemma If $\Gamma =$, $\Gamma' =$, $\Gamma'' =$



then $d(\Gamma) = 2d(\Gamma') - 4\cos^2(\frac{\pi}{m}) d(\Gamma'')$

Pf. $A_\Gamma =$



expand by minors ∎

So:

- $d(A_n) = 2d(A_{n-1}) - d(A_{n-2})$

  $d(A_1) = \det\boxed{2} = 2$
  $d(A_2) = \det\begin{bmatrix}2&1\\1&2\end{bmatrix} = 3$ $\rightarrow d(A_n) = n+1$

- $d(B_n) = 2d(B_{n-1}) - d(B_{n-2})$ $\quad n \geq 4$

  $d(B_2) = 2$
  $d(B_3) = 2d(B_2) - d(A_1) = 2$ $\rightarrow d(B_n) = 2$

- $d(D_n) = 2d(D_{n-1}) - d(D_{n-2})$ $\quad n \geq 6$

  $d(\succ) = 2d(>) - d(:) = 2(4) - 2\cdot 2$
  $d(\succ\!-) = 2d(\succ) - d(>) = 2(4) - 4$ $\rightarrow d(D_n) = 4$

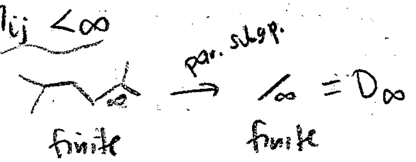
Similarly,

- $d(E_6) = 3$
- $d(E_7) = 2$
- $d(E_8) = 1$
- $d(F_4) = 1$
- $d(H_3) = 3 - \sqrt{5} > 0$
- $d(H_4) = \frac{7 - 3\sqrt{5}}{2} > 0$
- $d(I_2(m)) = 4\sin^2\frac{\pi}{m} > 0$ ∎

(II) If W is finite irred, then Γ is on the list

Sup W is finite. Then:

1. $m_{ij} < \infty$



✓

2. Γ has no cycles.

If it did, take an induced cycle