

PF

Induct. $|x| \rightarrow \text{clear}$

Sup true for $(n-1) \times (n-1)$. Let $A = \boxed{A'}$

\Downarrow : A' is pos. def because

$$\begin{array}{|c|} \hline x' \\ \hline \end{array} \begin{array}{|c|} \hline A' \\ \hline \end{array} \begin{array}{|c|} \hline x \\ \hline \end{array} = \begin{array}{|c|} \hline x' \\ \hline \end{array} \begin{array}{|c|} \hline A \\ \hline \end{array} \begin{array}{|c|} \hline x \\ \hline \end{array} > 0$$

So by induction, first $n-1$ ppal minors > 0

last one is $\det A = \lambda_1 \dots \lambda_n > 0$. \square

\Uparrow : The $(n-1)$ -dim subspace $U = \left\{ \begin{array}{c} x \\ 0 \end{array} \right\}$

has $x^T A x = x^T A' x > 0$.

let an orthonormal basis of eigenvectors for A be

$$v_1, \dots, v_k, v_{k+1}, \dots, v_n$$

$$\lambda_1, \dots, \lambda_k \leq 0 \quad \lambda_{k+1}, \dots, \lambda_n > 0$$

Then $\text{span}(v_1, \dots, v_k)$ has $x^T A x \leq 0$

Since these two don't intersect, $k \leq 1$

so A has at most one $\lambda \leq 0$.

But $\det A = \lambda_1 \dots \lambda_n > 0$ so I can't

have any $\lambda \leq 0$. \square

By Sylvester's criterion, it is time to compute some determinants!

Given a Coxeter graph 


when is the matrix $A = \left[-\cos\left(\frac{\pi}{m_{ij}}\right) \right]_{1 \leq i, j \leq n}$

positive definite?


Call these "pos def" graphs, they correspond to the finite Coxeter groups.

Theorem The indecomposable finite Coxeter groups are:

A_n 

B_n 

D_n 

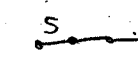
E_6 

E_7 

E_8 

F_4 

H_3 

H_4 

$I_2(m)$ 