Theorem.

(a) If $\lambda \in V$ then the subgroup of W that fixes λ is generated by the reflection s_α, ($\alpha \in \Phi$) which fix λ.

(b) Same for the subgp of W that fixes $U \subset V$ pointwise.

Pf.

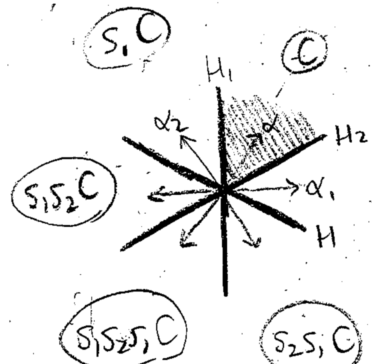
(a) We know this for $\lambda \in D$.

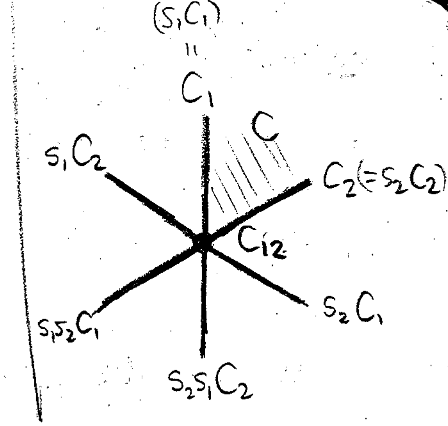
For others let $\lambda = w\mu$.

u fixes λ iff $u\lambda = \lambda$

$$u w \mu = w\mu$$
$$w^{-1} u w \mu = \mu$$
$$w^{-1} u w = s_1 \cdots s_k \qquad s_i \mu = \mu$$
$$u = (w s_1 w^{-1}) \cdots (w s_k w^{-1})$$
$$w s_i w^{-1} \lambda = \lambda$$

(b) Easy induction. ∎





- The hyperplane arrangement $\{H_\alpha\}_{\alpha \in \Pi}$ divides V into chambers (connected components of $V \setminus \bigcup_{\alpha \in \Pi} H_\alpha$)

- The fundamental chamber C has walls $H_{\alpha_1}, \ldots, H_{\alpha_n}$

- The other chambers are wC ($w \in W$)

$\boxed{\text{Ex: } \ell(w) = \#\text{ of hyperplanes separating } C, wC}$

Combin of Fundamental Domain

$$C_I = \left\{ \lambda \in D \;\middle|\; \begin{matrix} (\lambda,\alpha) = 0 & \alpha \in \Delta_I \\ (\lambda,\alpha) > 0 & \alpha \notin \Delta_I \end{matrix} \right\}$$