- **#5** Give an example of an infinite Coxeter group whose weak order has no infinite anti-chains, and an example of an infinite Coxeter group whose weak order has an infite anti-chain.

Proof. The Coxeter group generated by $S = \{a, b\}$ with $m(a, b) = \infty$ has no anit-chains of more than two elements since any $w \in (W, S)$ is either in the chain $e \leq_R a \leq_R ab \leq_R aba...$ or $e \leq_R b \leq_R ba \leq_R bab...$ and given three elements two will atleast two will be in the same chain.

The Coxeter group generated by $S = \{a, b, c\}$ with $m(a, b) = \infty, m(a, c) = \infty$, and $m(b, c) = \infty$ has infite anti-chains. For example, $\{b, ab, acb, acab, acacb, acacab. \dots\}$.
$\square$