3. We will divide the proof into several steps.

- **Step 1:** Let $x, y \in W$, and suppose $s \in S$ is such that $s \not\leq_R x$ and $s \not\leq_R y$. Then $s \not\leq_R x \vee y$.

  Suppose on the contrary that $s \leq_R x \vee y$. Since $s \not\leq_R x$ then $l(sx) = l(x) + 1$. We also have that $sx \leq_R w_0$, and thus $x = s(sx) \leq_R sw_0$. Similarly we get $y \leq_R sw_0$, so $x \vee y \leq_R sw_0$. Then $s \leq_R sw_0$, but this is a contradiction because $l(ssw_0) = l(w_0) > l(sw_0)$.

- **Step 2:** If $w \in W$ then the set $A_w = \{x \in W : x \wedge w = e \text{ and } x \vee w = w_0\}$ is closed under joins.

  Take $x, y \in A_w$. It is clear that $(x \vee y) \vee w = w_0$. If $(x \vee y) \wedge w >_R e$ then there exists an $s \in S$ such that $(x \vee y) \wedge w \geq_R s$. Now, since $x \in A_w$ and $s \leq_R w$ then $s \not\leq_R x$ (otherwise $s \leq_R x \wedge w$). Similarly $s \not\leq_R y$, so by Step 1 we have that $s \not\leq_R x \vee y$, which is a contradiction.

- **Step 3:** If $w \in W$ then the set A_w is closed under meets.

  Since multiplication on the left by w_0 is an antiautomorphism of the weak order then it takes meets to joins and viceversa, so

$$\begin{aligned} A_w &= \{x \in W : x \wedge w = e \text{ and } x \vee w = w_0\} \\ &= \{x \in W : w_0(x \wedge w) = w_0 \text{ and } w_0(x \vee w) = e\} \\ &= \{x \in W : w_0 x \vee w_0 w = w_0 \text{ and } w_0 x \wedge w_0 w = e\} \\ &= \{w_0 y \in W : y \vee w_0 w = w_0 \text{ and } y \wedge w_0 w = e\} \\ &= w_0 A_{w_0 w}, \end{aligned}$$

  which is closed under meets by Step 2.

- **Step 4:** If $w \in W$ then the set A_w is an interval.

  By steps 2 and 3 we have that A_w has a least element $u = \bigwedge A_w$ and a greatest element $v = \bigvee A_w$. Therefore $A_w = [u, v]$, because if $u \leq_R x \leq_R v$ then $x \wedge w \leq_R v \wedge w = e$ and $x \vee w \geq_R u \vee w = w_0$, so $x \in A_w$.