

9. Assume by contradiction that there exists an infinite antichain in the Bruhat order of W . We will say that a sequence w_1, w_2, w_3, \dots of elements in W is “good” if $w_j \not\leq w_k$ for $j < k$. In particular every antichain is a “good” sequence. Lets construct a “good” sequence as follows: Let w_1 be an element of least length among all first elements of all “good” sequences. Let w_2 be an element of least length among all second elements of all “good” sequences starting with w_1 . Let w_3 be an element of least length among all third elements of all “good” sequences starting with w_1, w_2 . Continue this process up to infinity, so we get a “good” sequence w_1, w_2, w_3, \dots . Fix reduced expressions for all the elements in this “good” sequence. Since there are only finitely many elements in S , there must be infinitely many of these expressions that start with the same letter $s \in S$, say $w_{i_1} = sw'_{i_1}, w_{i_2} = sw'_{i_2}, w_{i_3} = sw'_{i_3}, \dots$ with $i_1 < i_2 < i_3 < \dots$. Then by the subword property we have that $w'_{i_j} \not\leq w'_{i_k}$ for $j < k$. Then, again by the subword property, we have that $w_1, w_2, w_3, \dots, w_{i_1-1}, w'_{i_1}, w'_{i_2}, w'_{i_3}, \dots$ is a “good” sequence. But $l(w'_{i_1}) = l(w_{i_1}) - 1$, contradicting the choice of w_{i_1} .