8 Knowing the Bruhat order for (W, S) gives us in particular its graph of covering relations, and each of these covering relations corresponds to a single deletion and thus is an edge in the Bruhat graph. So our task is to reconstruct the edges (w, w') of the Bruhat graph with $l(w') - l(w) > 1$. Any Bruhat edge joins two comparable elements, so it's enough to look at each interval $[w, w']$ in the Bruhat order and determine whether the edge (w, w') should exist. By induction we can assume we've already done so for all proper subintervals of $[w, w']$.

We claim that, for any Bruhat interval $[w, w']$ with $l(w') - l(w) = d$, there are exactly d Bruhat edges from elements of this interval to w'. This will allow us to perform the reconstruction of the Bruhat graph: for each interval $[w, w']$, after having finished with its subintervals, w' will have either $d-1$ or d edges to it from within the interval; the edge (w, w') should be inserted if and only if there are $d-1$.

So, toward proving the claim, first take some reduced word $s_1 \ldots s_{l(w')}$ for w', so that it has a subword omitting only d letters $s_{i_1}, \ldots, s_{i_d}$ which is a reduced word for w. Omitting any single one of these letters s_{i_k} yields a word v with $w \leq v \leq w'$ such that an edge (v, w') exists in the Bruhat graph. This provides d edges overall; it remains to show there are no more.

Suppose this didn't hold; let $[w, w']$ be a counterexample with $l(w')$ minimal. There is a minimal word $s_1 \ldots s_{l(w')} = w'$, such that there are strictly more than d indices i such that deleting s_i from this word leaves a word $v \geq w$ in the Bruhat order. Let I be the set of these indices.

Consider now the element $w's$, where $s := s_{l(w')}$; this element satisfies $w's < w$. Given any $i \in I \setminus \{l(w')\}$, write $w_{\hat{i}}$ for the word obtained by deleting i from w'. This is a reduced word, and it ends in s, so $w_{\hat{i}}s < w_{\hat{i}}$.

Now, we have two cases, according to whether $l(w') \in I$, equivalently whether $ws < w$ or $ws > w$. If $ws > w$, then the subword of $s_1 \ldots s_{l(w')}$ giving w omits $s_{l(w')}$, so that for any $i \in I \setminus \{l(w')\}$, $w_{\hat{i}}s$ is still a superword of w. There are more than $d-1$ elements of $I \setminus \{l(w')\}$, and thus the interval $[w's, w]$ of length $d-1$ constitutes a smaller counterexample. If instead $ws < w$, then for every $i \in I$, we have $w_{\hat{i}}s < ws$ by lifting, and there are more than d such indices i, so the interval $[w's, ws]$ of length d is a smaller counterexample. In either case we have a contradiction.