9 For compactness, given a word $w = s_1 \cdots s_n \in S$, we shall abbreviate it by $w = s_{1..n}$, and likewise any subword $s_k \cdots s_l$ by $s_{k..l}$.

Let (W, S) be the Coxeter system for the group. For any reflection t and word $w = s_{1..n}$, define

$$\operatorname{sgn}(w,t) = (-1)^{\#\{k \,:\, 1 \le k \le n, t = s_{1..k} s_{1..k-1}^{-1}\}}$$

as in our discussion of the signed permutation representation. By the well-definedness of that representation, $\operatorname{sgn}(w,t)$ is well-defined if w is considered simply to be an element of W, rather than a particular word over S.

So, let t be a reflection. We then claim that $\operatorname{sgn}(t,t) = -1$. To see this, consider a word of form wsw^{-1} for t. This word has the two consecutive prefixes w and ws with $t = wsw^{-1}$; this contributes -1 to the sign. Suppose there were another such expression, i.e. a decomposition of the word wsw^{-1} as $xs'y$ for words x, y and $s' \in S$, with $t = xs'x^{-1}$. Then $y = (xs')^{-1}t = x^{-1}$, and so $t = y^{-1}s'y$ as well, which is an expression of t as the ratio of the two consecutive suffixes $s'y$ and y. But, since t is palindromic, these consecutive suffixes can be seen as consecutive prefixes y^{-1} and $(s'y)^{-1}$, and this pair is different to our first pair x and xs' since s' was not the middle character of wsw^{-1} by assumption. So these two expressions contribute 1 to $\operatorname{sgn}(t,t)$, i.e. cancel each other, and there will only be the one non-cancelling expression $t = wsw^{-1}$ we remarked on earlier. So $\operatorname{sgn}(t,t) = -1$ as claimed.

Now, let $s_{1..n}$ be any reduced word for t. Since $\operatorname{sgn}(t,t) = 1$ there must be at least one index k such that $s_{1..k}s_{1..k-1}^{-1} = t = s_{1..k}s_{k+1..n}$, implying $s_{1..k-1}^{-1} = s_{k+1..n}$. Then, in our word $s_{1..n}$, we can thus either replace the prefix $s_{1..k-1}$ by $s_{k+1..n}^{-1}$ or, oppositely, $s_{k+1..n}$ by $s_{1..k-1}^{-1}$, in both cases obtaining another word for t. Both the results of these replacements are palindromes, and at least one is at least as short as $s_{1..n}$ and is therefore reduces. We conclude t has a palindromic reduced expression.