7. Let $M = \{\dim(E_i \cap F_j)\}_{1 \le i,j \le n}$, and let $j \in [n]$. The numbers in the j-th column of M correspond to the sequence

$$\dim(E_1 \cap F_j), \dim(E_2 \cap F_j), \ldots, \dim(E_n \cap F_j),$$

which is clearly a non-decreasing sequence. Note that the kernel of the canonical homomorphism

$$f : E_i \cap F_j \to E_i/E_{i-1}$$

is $\ker(f) = (E_i \cap F_j) \cap E_{i-1} = E_{i-1} \cap F_j$, so it induces an injection

$$\hat{f} : (E_i \cap F_j)/(E_{i-1} \cap F_j) \hookrightarrow E_i/E_{i-1}.$$

Therefore

$$\dim(E_i \cap F_j) - \dim(E_{i-1} \cap F_j) = \dim\left((E_i \cap F_j)/(E_{i-1} \cap F_j)\right) \le \dim(E_i/E_{i-1}) = 1,$$

which shows that the sequence of numbers in the j-th column of M increases at most 1 at each step. Define

$$S_j = \{i \in [n] : \dim(E_i \cap F_j) - \dim(E_{i-1} \cap F_j) = 1\},$$

that is, S_j is the set of steps where the sequence of numbers in the j-th column increases. Since $\dim(E_0 \cap F_j) = \dim(0) = 0$ and $\dim(E_n \cap F_j) = \dim(F_j) = j$ then the set S_j contains exactly j elements. Moreover, note that if $j > 1$ and $i \notin S_j$ then

$$\dim(E_i \cap F_j) - \dim(E_{i-1} \cap F_j) = 0$$

and so $E_i \cap F_j = E_{i-1} \cap F_j$, thus

$$E_i \cap F_{j-1} = (E_i \cap F_j) \cap F_{j-1} = (E_{i-1} \cap F_j) \cap F_{j-1} = E_{i-1} \cap F_{j-1}$$

and

$$\dim(E_i \cap F_{j-1}) - \dim(E_{i-1} \cap F_{j-1}) = 0,$$

showing that $i \notin S_{j-1}$. We have then that $S_1 \subseteq S_2 \subseteq \cdots \subseteq S_n$. Since for all j we have that $\#S_j = j$, then for all j there is a unique number $\sigma(j) \in S_j \setminus S_{j-1}$ (where we take $S_0 = \emptyset$). Note that $\{\sigma(1), \sigma(2), \ldots, \sigma(n)\} = S_n = [n]$, so it is clear that σ is a permutation of the set $[n]$. Moreover, M is the rank table of the permutation σ, because for any pair i, j we have that the number of dots in the northwest corner above the point (i, j) in the rank table of σ is

$$\begin{aligned}
\#\{k \leq j : \sigma(k) \leq i\} &= \#\{k \leq j : S_k \setminus S_{k-1} \leq i\} \\
&= \#\left(\left(\left(S_1 \setminus S_0\right) \cup \left(S_2 \setminus S_1\right) \cup \cdots \cup \left(S_j \setminus S_{j-1}\right)\right) \cap [i]\right) \\
&= \#(S_j \cap [i]) \\
&= M_{i,j}
\end{aligned}$$

directly from the definition of S_j.