1. (*Sketch.*) Let G be the group of symmetries in question. It is easy to check that the three reflections a, b, c (as labelled in Amanda's beautiful picture) across the three edges of a "base" unit triangle satisfy the Coxeter relations of the Coxeter group $\widetilde{A}_2$ whose diagram is the unlabelled triangle. Also a, b, c generate G, since the elements of G are in bijection with the unit triangles of the grid, and the base triangle can be sent to every other triangle in the grid. This provides a labelling of the triangles of the grid with elements of $\widetilde{A}_2$.

To show that this labelling induces a bijection between the groups G and $\widetilde{A}_2$ we count that they have the same number of elements of each length. As Mariana's beautiful picture clearly shows, there are $3k$ elements of length k in G. One then needs to prove that $\widetilde{A}_2$ also has exactly $3k$ elements of length k, which can be checked in various ways - one way is deducing it from the Poincaré series, which we computed to be

$$\widetilde{A}_2(q) = \frac{1 + q + q^2}{(1 - q)^2}$$