

Problem 10. Let $S = \{s_1 s_n, s_2 s_n, \dots, s_{n-1} s_n\}$. Let w be a word in the alternating group of W , then $w = s_{m_1} s_{m_2} s_{m_3} \dots s_{m_k}$ with k even and we could group the expression in pairs as,

$$w = (s_{m_1} s_{m_2})(s_{m_3} s_{m_4}) \dots (s_{m_{k-1}} s_{m_k})$$

Consider a pair $(s_{m_{2i-1}} s_{m_{2i}})$. The case where $m_{2i-1} = m_{2i}$ is trivial, so suppose $m_{2i-1} \neq m_{2i}$. If $m_{2i} = n$, then $(s_{m_{2i-1}} s_{m_{2i}}) \in S$. If $m_{2i-1} = n$, then

$(s_{m_{2i-1}}s_{m_{2i}}) = (s_{m_{2i}}s_{m_{2i-1}})^{-1}$ and $(s_{m_{2i}}s_{m_{2i-1}}) \in S$, so the pair is generated by S . Finally, suppose $m_{2i-1}, m_{2i} \neq n$, then we can write the pair as $(s_{m_{2i-1}}s_{m_{2i}}) = (s_{m_{2i-1}}s_n)(s_{m_{2i}}s_n)^{-1}$, so that pair is also generated by S . As the word w is finite, I could do that with every pair to conclude that w is generated by S . Now, it's clear that every word in W formed by the elements of S has an even number of words, so it's length is even too and it belongs to the alternating group.