

1. Suppose the Coxeter diagram of  $(W, \{a, b, c\})$  has an unlabelled edge between  $a$  and  $b$ , an edge labeled  $m$  between  $b$  and  $c$ , and no edge between  $a$  and  $c$ . If the relations  $bcbcacababcacbcabacbabacbc=e$  holds, determine  $m$ .

$W$  is a group so associativity holds. And we are given the following additional relations

$$e = (bc)^m = a^2 = b^2 = c^2 = (ab)^3$$

$$aba = bab$$

$$ac = ca$$

$$\begin{aligned} e &= (bc)b(caca)(bab)cacbcabac(bab)acbc = (bc)b(c(aa)c)(aba)(cac)bcabac(aba)acbc \\ &= (bc)b(cc)(aba)a(cc)bcabacab(aa)cbc = (bc)bab(aa)bcabacabc = (bc)ba(bb)(ca)ba(ca)bcbc \\ &= (bc)ba(ac)ba(ac)(bc)(bc) = (bc)b(aa)cb(aa)c(bc)(bc) = (bc)(bc)(bc)(bc)(bc) = (bc)^5 \end{aligned}$$

5 is minimal because nothing but 1 divides into it. However  $bc$  is not the identity so the order is not 1.

This means  $m = 5$