1 The fundamental relationship we know between the h- and f-polynomials is

$$h(t) = (1-t)^d f\left(\frac{t}{1-t}\right).$$

Skandera's trick is essentially a direct restatement of this, in view of the fact that for a polynomial g of degree d, reversing the list of coefficients of $g(t)$ yields $t^d g(t^{-1})$. So the steps of the trick successively yield $t^d f(t^{-1})$, then $(t-1)^d f((t-1)^{-1})$, then $t^d(t^{-1}-1)^d f((t^{-1}-1)^{-1})$, which in view of $t^{-1}-1 = (1-t)/t$ simplifies to $(1-t)^d f(t/(1-t)) = h(t)$.

Having this, we need only show that Stanley's trick carries the polynomial $g(t)$ on the falling diagonal to the polynomial $g(t-1)$ read left-to-right across the bottom. We show this inductively on the degree d of g. Let $g_d, \ldots, g_0$ be the coefficients of $g(t)$, and $g'_d, \ldots, g'_0$ those of $g(t-1)$. If $d = 0$, then $g_0 = g'_0$ and the array is simply

$$\begin{array}{cc} & g_0 \\ & \\ g_0 & \end{array}$$

Otherwise let $k(t) = (g(t) - g(0))/t$ be the polynomial obtained by truncating the last coefficient of g, and define the coefficients k_i and k'_i in the obvious fashion. By induction we have an array of form

$$\begin{array}{ccccccc} & & & k_{d-1} & & & \\ & & \iddots & & \ddots & & \\ & \iddots & & \iddots & & k_0 & \\ k'_{d-1} & & \cdots & & k'_0 & & \end{array}$$

Now suppose we add the coefficient g_0 to the space in the lower-right corner, and compute another row of differences along the bottom. In the ith position we get $k'_i - k'_{i+1}$, with the convention $k'_d = 0$ and $k'_{-1} = g_0$. and we want to prove this equal to h'_{i+1} for the induction to go through. By expanding and using the fact that the k' are the coefficients of $k(t-1)$, we can check that these differences are the coefficients of $(t-1)k(t-1) + g_0$, which by the construction of k is $g(t-1)$. This completes the induction.