2. **(a)** Let $deg(P) = m$ so dividing by $\binom{x}{m}$, then the residue by $\binom{x}{m-1}$ and so on (we ignore $\binom{x}{j}$ with $j > m$ because they have bigger degree) we get that $P = c_0\binom{x}{0} + c_1\binom{x}{1} + \cdots + c_m\binom{x}{m}$ for some c_i's reals. Evaluating at $x = 0$ we get that c_0 is integer. Then with $x = 1$ $c_0 + c_1$ is integer, because the rest of polynomials are zero in 1, so c_1 is integer. Then by induction suppose that until $i = k$, the c_i's are integers, now we evaluate at $k+1$. $\binom{k+1}{j}$ is integer for all j integer, it is zero for $j > k+1$ and it is one if $j = k+1$, so because the polynomial is intger in $k+1$, that integer should be equal to a sum of integers (remember c_i for $i < k+1$ are integers) plus the single term c_{k+1}, and it becomes clear that c_{k+1} is intger. So c_i's are integers. Repeating the above argument works similarly to show that if we have the zero polynomial, each coefficient must be 0 (by first evaluating in 0, then in 1, and so on). So they form a basis and we have a free abelian group.

   (b) No. Consider the ideal of polynomials that are integer when evaluated at integer and that in zero they are zero, that is, they don't have a constant term. Suppose it is generated by a finite combinations of the $\binom{x}{k}$, for some k. That means that the polynomial $\binom{x}{p}$ for a prime p larger than any of the k's appearing in the basis, is a combination of elements in the base. However if that was the case, evaluating at p we get 1 at one side of the equation, and something multiple of p in the other side (since $\binom{p}{k}$ is divisible by p por all $0 < k < p$ and we don't have constant term so we dont have $\binom{x}{0}$ in the generating set), so we have a contradiction. That ideal can't be finitely generated so the ring is not noetherian.