1a. Let S be any subset of P, and let M be the set of minimal monomials of S. Assume by contradiction that M is infinite (but of course, countable), so we can write $M = \{m_1, m_2, m_3, \ldots\}$. Consider the graph G whose vertices are the monomials in M, and with an edge between any two distinct vertices. We are going to color the edges of G using the colors $\{1, 2, \ldots, n\}$ in the following way: If $i < j$ then the monomials m_i and m_j are not comparable by divisibility (since they are both minimal elements of S), so there exists some index $c \in \{1, 2, \ldots, n\}$ such that the exponent of the variable x_c in m_i is greater than the exponent of x_c in m_j (otherwise m_i would divide m_j). We will color the edge between m_i and m_j of this color c.

Now, we have an infinite complete graph whose edges are colored with finitely many colors, so by the Infinite Ramsey Theorem (see http://en.wikipedia.org/wiki/Ramsey's_theorem) we know that there is an infinite monochromatic complete subgraph, that is, there is a subset $N = \{m_{i_1}, m_{i_2}, m_{i_3}, \ldots\}$ of M such that all the edges between elements of N have the same color c_0. But then, by the definition of our coloring, this means that the exponents of x_{c_0} in the monomials $m_{i_1}, m_{i_2}, m_{i_3}, \ldots$ form an infinite decreasing sequence of non-negative integers, which is of course a contradiction.